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Decision Support

The evidential reasoning approach for multiple attribute decision analysis using interval belief degrees

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Abstract

Multiple attribute decision analysis (MADA) problems having both quantitative and qualitative attributes under uncertainty can be modeled using the evidential reasoning (ER) approach. Several types of uncertainties such as ignorance and fuzziness can be modeled in the ER framework. In this paper, the ER approach will be extended to model new types of uncertainties including interval belief degrees and interval data that could be incurred in decision situations such as group decision making. The Dempster–Shafer (D–S) theory of evidence is first extended, which is one of the bases of the ER approach. The analytical ER algorithm is used to combine all evidence simultaneously. Two pairs of nonlinear optimization models are constructed to estimate the upper and lower bounds of the combined belief degrees and to compute the maximum and the minimum expected utilities of each alternative, respectively. Interval data are equivalently transformed to interval belief degrees and are incorporated into the nonlinear optimization models. A cargo ship selection problem is examined to show the implementation process of the proposed approach.

Keywords: Multiple attribute decision analysis; The evidential reasoning approach; Uncertainty modeling; Interval degrees of belief; Interval data; Nonlinear optimization

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1. Introduction

Many complex multiple attribute decision analysis (MADA) problems involve both quantitative and qualitative attributes as well as various types of uncertainties such as incomplete information, complete ignorance and fuzziness. Such complex MADA problems can always be modeled using the evidential reasoning (ER) approach (Yang and Sen, 1994; Yang and Singh, 1994; Yang, 2001; Yang and Xu, 2002a,b; Xu et al., in press; Wang et al., in press; Yang et al., in press). The ER approach models both quantitative and qualitative attributes using a distributed modeling framework, in which each attribute is characterized by a set of collectively exhaustive assessment grades, probabilistic uncertainty including incomplete information and complete ignorance by a belief structure, and fuzzy uncertainty by fuzzy linguistic variables.

In certain decision situations such as group decision making, however, a new type of interval uncertainty is likely encountered. For example, quantitative data may not be known precisely but may be estimated to belong to intervals with certain confidence levels. A decision maker (DM) may be unable to give precise judgement. In group decision analysis, different DMs may assign different degrees of belief to the same judgment. It will be very difficult to synthesize different degrees of belief to generate a precise point estimate if DMs cannot reach a consensus. Using interval belief degree may be a sensible option in such circumstances.

Xu et al. (in press) investigated the ER approach for MADA under interval uncertainties. They looked into another type of interval uncertainty caused by interval assessment grades. For instance, in real decision analyses, some alternative(s) may not be assigned to some definite assessment grade, say, *Excellent* or *Very Good* or *Good*. In this situation, DM may prefer to assign it/them to an interval assessment grade, say between *Excellent* and *Good*. It can be either *Excellent* or *Very Good* or *Good*. But the DM may not be sure which one.

The purpose of this paper is to investigate the first two types of interval uncertainties caused by interval data and interval belief degrees and to develop the ER approach for MADA with these two types of interval uncertainties. Due to the presence of interval belief degrees, the original Dempster–Shafer (D–S) theory of evidence, which is one of the bases of the ER approach, needs to be extended. The focus is on combining and normalizing interval evidence. This is because the combination and normalization process of interval evidence no longer preserves the associative property that the process does not depend on the order in which evidence is combined. To preserve the property, the analytical ER algorithm is used to combine all evidence simultaneously before the combined belief degrees are normalized which are of an interval nature as well. A pair of nonlinear optimization models is constructed to estimate the upper and lower bounds of the combined belief degrees. Interval data are equivalently transformed to interval belief degrees and are incorporated into the nonlinear optimization models.

The paper is organized as follows. In Section 2, the original D–S theory of evidence is first extended to combine interval evidence and the relevant theoretical issues are discussed. Section 3 develops the ER approach for MADA with interval data and interval belief degrees, where interval data are transformed into interval belief degrees, the analytical ER algorithm is utilized to combine all evidence simultaneously, two pairs of nonlinear optimization models are constructed to estimate the upper and lower bounds of the combined belief degrees and to compute the maximum and the minimum expected utilities of each alternative, respectively. Section 4 provides a real numerical example to illustrate the application and the detailed implementation process of the proposed approach. The paper is concluded in Section 5. A minimax regret approach (MRA) for ranking interval-valued expected utilities is provided in Appendix A.