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Decision Support

Simulation of fuzzy multiattribute models for grey relationships

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Abstract

Multiattribute decision making involves tradeoffs among alternative performances over multiple attributes. The accuracy of performance measures are usually assumed to be accurate. Most multiattribute models also assume given values for the relative importance of weights for attributes. However, there is usually some uncertainty involved in both of these model inputs. Outranking multiattribute methods have always provided fuzzy input for performance scores. Many analysts have also recognized that weight estimates also involve some imprecision, either through individual decision maker uncertainty, or through aggregation of diverging group member preferences. Many fuzzy multiattribute models have been proposed, but they have focused on identifying the expected value solution (or extreme solutions). This paper demonstrates how simulation can be used to reflect fuzzy inputs, which allows more complete probabilistic interpretation of model results.

Keywords: Multiple attribute decision making (MADM); Fuzzy sets; Monte Carlo simulation; Grey analysis

1. Introduction

Multiattribute decision making has progressed in a variety of directions throughout the world. Most models are deterministic, to include multiattribute utility theory [8] and AHP [14]. Outranking methods from various schools [9] also support deterministic inputs, although methods such as ELECTRE [12] and PROMETHEE [2] have always supported fuzzy input for alternative performances on attributes.

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Recognition that real life decisions involve high levels of uncertainty is reflected in the development of fuzzy multiattribute models. Fuzzy methods have been widely published in multiattribute decision making [1,3] to include AHP [7]. Uncertain input in the form of rough sets has also been proposed [16]. The method of grey analysis [4] is another approach to reflecting uncertainty in the basic multiattribute model:

$$\text{value}_j = \sum_{i=1}^{K} w_i \times u(x_{ij}), \tag{1}$$

where w_i is the weight of attribute i, K is the number of attributes, and $u(x_{ij})$ is the score of alternative x_j on attribute i.

Grey system theory was developed by Deng [4], based upon the concept that information is sometimes incomplete or unknown. The intent is the same as with factor analysis, cluster analysis, and discriminant analysis, except that those methods often do not work well when sample size is small and sample distribution is unknown [15]. Interval numbers are standardized through norms, which allow transformation of index values through product operations.

This paper addresses the use of Monte Carlo simulation to this model to reflect uncertainty as expressed by fuzzy input. Simulation has been applied to AHP [10], generating random pairwise comparison input values. Our paper differs from past papers in that instead of estimating the expected value or extreme performance of alternatives, simulation offers a more complete understanding of the possible outcomes of alternatives as expressed by fuzzy numbers. The focus is on probability rather than on maximizing expected or extreme values. Both weights and alternative performance scores are allowed to be fuzzy. Both interval and trapezoidal fuzzy input are considered.

2. Grey related analysis

Grey related analysis is a technique that can be applied to both fuzzy and crisp data. Classical grey related analysis is based upon time series data and/or cross-sectional data [11]. This paper extends that approach to a multiattribute decision making context. We present it here as a means to obtain a solution from fuzzy data. Suppose that a multiple attribute decision making problem with interval numbers has m feasible plans $X_1, X_2, \ldots, X_m$, with n indexes. The weight value w_j of index G_j is uncertain, but $w_j \in [c_j, d_j]$, $0 \leqslant c_j \leqslant d_j \leqslant 1$, $j = 1, 2, \ldots, n$, $w_1 + w_2 + \cdots + w_n = 1$, and the index value of jth index G_j of feasible plan X_i is an interval number $[a_{ij}^-, a_{ij}^+]$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$. When $c_j = d_j$, $j = 1, 2, \ldots, n$, the multiple attribute decision making problem with interval numbers is called a multiple attribute decision making problem with interval-valued indexes. When $a_{ij}^- = a_{ij}^+$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, the multiple attribute decision making problem with interval numbers is called a multiple attribute decision making problem with interval-valued weights. The principle and steps of the grey related analysis method are as follows:

Step 1: Construct decision matrix A with index number of interval numbers

If the index value of jth index G_j of feasible plan X_i is an interval number $[a_{ij}^-, a_{ij}^+]$, $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, decision matrix A with index number of interval numbers is defined as the following:

$$A = \begin{bmatrix} [a_{11}^-, a_{11}^+] & [a_{12}^-, a_{12}^+] & \ldots & [a_{1n}^-, a_{1n}^+] \\ [a_{21}^-, a_{21}^+] & [a_{22}^-, a_{22}^+] & \ldots & [a_{2n}^-, a_{2n}^+] \\ \ldots & \ldots & \ldots & \ldots \\ [a_{m1}^-, a_{m1}^+] & [a_{m2}^-, a_{m2}^+] & \ldots & [a_{mn}^-, a_{mn}^+] \end{bmatrix}. \tag{2}$$