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Innovative Applications of O.R.

Cut generation for an integrated employee timetabling and production scheduling problem

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ABSTRACT

This paper investigates the integration of the employee timetabling and production scheduling problems. At the first level, we manage a classical employee timetabling problem. At the second level, we aim at supplying a feasible production schedule for a set of interruptible tasks with qualification requirements and time-windows. Instead of hierarchically solving these two problems as in the current practice, we try here to integrate them and propose two exact methods to solve the resulting problem. The former is based on a Benders decomposition while the latter relies on a specific decomposition and a cut generation process. The relevance of these different approaches is discussed here through experimental results.

1. Introduction

The ultimate purpose of any production system is merely to produce goods to meet some demand. To do so, one must find a production schedule, that is an allocation of human and material resources to the different tasks (or jobs) that have to be processed. However, such a schedule has to take into account the availability of the resources, in particular the human ones: this means that an appropriate employee timetable has to be built up simultaneously to the production schedule. When the production is intended to meet a given, fixed demand, production costs (raw material, energy, etc.) do not vary significantly, whatever the actual schedule is, with regard to labor costs. It is hence reasonable to consider that employee timetabling should aim at minimizing labor costs while production scheduling should insure that the production can actually be done on time.

Although an integrated approach should clearly be adopted when looking for a global optimum, the resulting problem is usually considered as too complex to be solved in practice, and it is decomposed into an assignment part and a scheduling part. As a consequence, even though there is a huge literature on both scheduling problems (e.g., see Pinedo [16] and Leung [14]) and timetabling problems (see Ernst et al. [8] and Soumis et al. [17] for states of the art), only few attempts exist for the integrated problem (see Artigues et al. [3] for an integrated approach and an exhaustive state of the art; see Hooker [11,12] for a hybrid method mixing Linear Programming and Constraint Programming).

In this paper, we exploit the ideas of Lasserre [13] and Dauzère-Pérès and Lasserre [6] for an integrated job-shop lot-sizing problem. These authors propose to solve the integrated problem alternatively at two different stages: either to compute lot-sizes for a sequence of jobs, or to compute a sequence for given lot-sizes. A similar decomposition approach has also been successfully applied by Detienne et al. to an employee timetabling problem with a given work load [7].

In the remainder of this paper, we propose two exact methods to solve the integrated employee timetabling and production scheduling problem. The problem description is given in Section 2, together with integer linear models. An exact method based on a Benders decomposition is addressed in Section 3 while the fourth section introduces an exact method based on a specific decomposition and cut generation process. We then discuss, in Section 5, about the relevance of these methods by comparing their computational results on generated instances. Some conclusions are finally drawn in Section 6.

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2. Problem description and MIP models

We want to schedule a set J of n independent jobs (tasks) with a set O of m resources (operators) over a planning horizon H. Each job $j \in J$ is characterized by a processing time p_j, a time-window $D_j = [r_j, d_j]$ and requires an operator $o \in O$ who masters one needed competence $c_j \in C$. Jobs may be interrupted and can be processed by different operators. However, processing instants of any given job cannot overlap in time. An operator cannot process several jobs simultaneously. Each operator $o \in O$ has a set $C_o \subseteq C$ of competences and owns a set $\Omega_o \subseteq \Omega$ of eligible work patterns. A work pattern $\omega \in \Omega$ defines a sequence of actual working time instants and breaks over the whole planning horizon. This formulation permits to take into account several contractual, legal or other constraints (vacations, individual preferences, etc.). Each relevant pair work pattern–operator (ω–o) is given a cost η^o_ω standing for the resulting labor cost of assigning work pattern ω to operator o.

Our problem consists in both scheduling the n jobs and assigning a work pattern to each operator in order to satisfy each need for workforce (number of operators and qualification requirements) at minimum cost.

Through this paper, we will consider a descriptive instance with 3 operators (o_1, o_2, o_3), 2 competences (c_1, c_2), 3 jobs (j_1, j_2, j_3) and 2 work patterns ($\omega_1 = [0, 8)$, $\omega_2 = [8, 16)$). Operators o_1 and o_2 only master competence c_1 whereas o_3 masters c_1 and c_2. The characteristics of this instance – Example 1 – are given in Table 1.

A feasible solution (cost: 26) for Example 1 consists in assigning ω_1 to o_1 and o_2 and ω_2 to o_3. The following Gantt chart (Fig. 1) illustrates the operations processed by operators.

2.1. Time-indexed formulation

y^o_ω is a binary decision variable where $y^o_\omega = 1$ if work pattern ω is assigned to operator o and $y^o_\omega = 0$ otherwise. Binary variable $x_{jt} = 1$ if and only if one unit of job j is processed at time instant t and binary variable $z_{oct} = 1$ if and only if operator o uses competence c at time t.

Any work pattern ω can be expressed by a boolean vector σ_ω over H such that $\sigma^t_\omega = 1$ if $t \in H$ is a working time instant and $\sigma^t_\omega = 0$ otherwise.

Using notations mentioned above, an intuitive MIP formulation can hence be given:

$$[Q] : \min \sum_{o \in O} \sum_{\omega \in \Omega_o} \eta^o_\omega \cdot y^o_\omega \tag{1}$$

$$\sum_{\omega \in \Omega_o} y^o_\omega = 1 \quad \forall o \in O, \tag{2}$$

$$\sum_{t \in D_j} x_{jt} = p_j \quad \forall j \in J, \tag{3}$$

$$\sum_{\substack{j \in J| \\ c_j = c}} x_{jt} = \sum_{\substack{o \in O| \\ c \in C_o}} z_{oct} \quad \forall t \in H,\ \forall c \in C, \tag{4}$$

$$\sum_{c \in C_o} z_{oct} \leqslant \sum_{\omega \in \Omega_o} \sigma^t_\omega \cdot y^o_\omega \quad \forall t \in H,\ \forall o \in O, \tag{5}$$

$$y^o_\omega \in \{0, 1\} \quad \forall o \in O,\ \forall \omega \in \Omega_o, \tag{6}$$

$$x_{jt} \in \{0, 1\} \quad \forall j \in J,\ \forall t \in H, \tag{7}$$

$$z_{oct} \in \{0, 1\} \quad \forall o \in O,\ \forall c \in C_o,\ \forall t \in H. \tag{8}$$

Table 1
Characteristics of Example 1.

	j_1	j_2	j_3		o_1	o_2	o_3
r_j	0	2	8	$\eta^o_{\omega_1}$	10	7	2
d_j	10	8	16	$\eta^o_{\omega_2}$	5	3	9
p_j	9	2	3				
c_j	c_1	c_1	c_2				

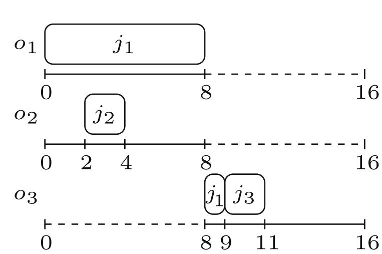

Fig. 1. Gantt chart.