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Discrete Optimization

The multi-depot vehicle routing problem with inter-depot routes

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Abstract

This article addresses an extension of the multi-depot vehicle routing problem in which vehicles may be replenished at intermediate depots along their route. It proposes a heuristic combining the adaptative memory principle, a tabu search method for the solution of subproblems, and integer programming. Tests are conducted on randomly generated instances.

Keywords: Multi-depot vehicle routing problem; Replenishment; Adaptative memory; Tabu search; Integer programming

1. Introduction

We study a variant of the multi-depot vehicle routing problem where depots can act as intermediate replenishment facilities along the route of a vehicle. This problem is a generalization of the *Vehicle Routing Problem* (VRP). The classical version of the VRP is defined on a graph $G = (V_c \cup V_d, A)$, where $V_c = \{v_1, v_2, \ldots, v_n\}$ is the customer set, $V_d = \{v_{n+1}\}$ is the depot set and $A = \{(v_i, v_j)\colon\ v_i, v_j \in V_c \cup V_d,\ i \neq j\}$ is the arc set of G. A fleet of m vehicles of capacity Q is located at v_{n+1}. Each customer has a demand q_i and a service duration d_i. A cost or travel time c_{ij} is associated with every arc of the graph. The VRP consists of determining m routes of minimal cost satisfying the following conditions: (i) every customer appears on exactly one route; (ii) every route starts and ends at the depot; (iii) the total demand of the

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customers on any route does not exceed Q; (iv) the total duration of a route does not exceed a preset value D.

Several algorithms are available for the VRP. Because this is a hard combinatorial problem, exact methods tend to perform poorly on large size instances, which is why numerous heuristics have been developed. These include *classical heuristics* such as construction and improvement procedures or two-phase approaches, and *metaheuristics* like simulated annealing, tabu search, variable neighborhood search and evolutionary algorithms. For surveys, see Laporte and Semet [22], Gendreau et al. [15] and Cordeau et al. [8].

In some contexts, one can assign more than one route to a vehicle. The *Vehicle Routing Problem with Multiple Use of Vehicles* (VRPM) is encountered, for example, when the vehicle fleet is small or when the length of the day is large with respect to the average duration of a route. Fleischmann [12] was probably the first to propose a heuristic for this problem. It is based on the savings principle for route construction combined with a bin packing procedure for the assignment of routes to vehicles. Taillard et al. [29] have developed an adaptative memory and a tabu search heuristic, again using a bin packing procedure for assigning routes to vehicles. Other heuristics have been proposed for the VRPM, such as those of Brandão and Mercer [3,4] or Zhao et al. [34] based on tabu search or, in the ship routing context, the methods proposed by Suprayogi et al. [28] and Fagerholt [11] which create routes by solving traveling salesman problems (TSPs) and solve an integer program (Suprayogi et al. proposed a set partitioning problem) for the assignment part.

Another well-known generalization of the VRP is the *Multi-Depot Vehicle Routing Problem* (MDVRP). In this extension every customer is visited by a vehicle based at one of several depots. In the standard MDVRP every vehicle route must start and end at the same depot. There exist only a few exact algorithms for this problem. Laporte et al. [20] as well as Laporte et al. [21] have developed exact branch-and-bound algorithms but, as mentioned earlier, these only work well on relatively small instances. Several heuristics have been put forward for the MDVRP. Early heuristics based on simple construction and improvement procedures have been developed by Tillman [30], Tillman and Hering [32], Tillman and Cain [31], Wren and Holliday [33], Gillett and Johnson [16], Golden et al. [17], and Raft [24]. More recently, Chao et al. [6] have proposed a search procedure combining Dueck's [10] record-to-record local method for the reassignment of customers to different vehicle routes, followed by Lin's 2-opt procedure [23] for the improvement of individual routes. Renaud et al. [26] described a tabu search heuristic in which an initial solution is built by first assigning every customer to its nearest depot. A petal algorithm developed by the same authors [25] is then used for the solution of the VRP associated with each depot. It finally applies an improvement phase using either a subset of the 4-opt exchanges to improve individual routes, swapping customers between routes from the same or different depots, or exchanging customers between three routes. The tabu search approach of Cordeau et al. [7] is probably the best known algorithm for the MDVRP. An initial solution is obtained by assigning each customer to its nearest depot and a VRP solution is generated for each depot by means of a sweep algorithm. Improvements are performed by transferring a customer between two routes incident to the same depot, or by relocating a customer in a route incident to another depot. Reinsertions are performed by means of the GENI heuristic [13]. One of the main characteristics of this algorithm is that infeasible solutions are allowed throughout the search. Continuous diversification is achieved through the penalization of frequent moves.

The *Multi-Depot Vehicle Routing Problem with Inter-Depot Routes* (MDVRPI) has not received much attention from researchers. A simplified version of the problem is discussed by Jordan and Burns [19] and Jordan [18] who assumed that customer demands are all equal to Q and that inter-depot routes consist of back-and-forth routes between two depots. The authors transform the problem into a matching problem which is solved by a greedy algorithm. Angelelli and Speranza [2] have developed a heuristic for a version of the *Periodic Vehicle Routing Problem* (PVRP) in which replenishments at intermediate facilities are allowed. Their algorithm is based on the tabu search heuristic of Cordeau et al. [7]. A version of the problem where