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Interfaces with Other Disciplines

On the estimation of returns-to-scale in FDH models

M. Soleimani-damaneh [*,1], G.R. Jahanshahloo, M. Reshadi

Faculty of Mathematical Sciences and Computer Engineering, Teacher Training University, 599 Taleghani avenue, Tehran 15618, Iran

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Abstract

This paper first reviews Kerstens and Vanden Eeckaut's (K&VE) method [Kerstens, K., Vanden Eeckaut, P., 1999. Estimating returns-to-scale using non-parametric deterministic technologies: A new method based on goodness-of-fit. European Journal of Operational Research 113, 206–214] for testing returns-to-scale (RTS) in free disposal hull (FDH) models. Then, an approach and an algorithm are introduced for this task, based on the evaluation of certain ratios of inputs and outputs, which have many computational advantages. Also, the equivalence between the proposed approach and K&VE method is proved.

Keywords: DEA; FDH; Returns-to-scale

1. Introduction

Free Disposal Hull (FDH) models were first formulated by Deprins et al. (1984). FDH relies on the sole assumption that production possibilities satisfy free disposability, and ensures that efficiency evaluations are effected from only actually observed performances.

Recently Kerstens and Vanden Eeckaut (K&VE) (1999) have introduced a method for testing returns-to-scale (RTS) in the FDH models. This method has been improved by Podinovski (2004). K&VE and Podinovski suggest solving three mixed integer programming problems and comparing

[*] Corresponding author. Tel.: +98 212093947/3482510017; fax: +98 217507772.
E-mail address: soleimani_d@yahoo.com (M. Soleimani-damaneh).
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the related efficiency scores for this task. Therefore, using these suggested approaches can be onerous from a computational point of view. Indeed, one must solve *3n* mixed integer problems for testing RTS of n DMUs.

After reviewing K&VE method in Section 2 of this paper, a new method with many computational advantages is introduced in Section 3. The introduced method categorizes DMUs into RTS classes based on the evaluation of certain ratios without solving any models.

2. Kerstens and Vanden Eeckaut's method

Assume that there are n DMUs, where each DMU_j $(j = 1, \ldots, n)$, uses m inputs, x_{ij} $(i = 1, \ldots, m)$, to produce s outputs, y_{rj} $(r = 1, \ldots, s)$. We shall also assume that $\mathbf{x}_j = (x_{1j}, \ldots, x_{mj}) \geqslant \mathbf{0}$, $\mathbf{y}_j = (y_{1j}, \ldots, y_{sj}) \geqslant \mathbf{0}$, $\mathbf{x}_j \neq \mathbf{0}$ and $\mathbf{y}_j \neq \mathbf{0}$, for each $j \in J = \{1, \ldots, n\}$. Define $\mathbf{X} = [\mathbf{x}_1, \ldots, \mathbf{x}_n]$, the $m \times n$ matrix of inputs, and $\mathbf{Y} = [\mathbf{y}_1, \ldots, \mathbf{y}_n]$, the $s \times n$ matrix of outputs.

The traditional FDH technology, from which the modeling of variable RTS (VRS) results, is as follows:

$$T^{\text{FDH}} = \{(\mathbf{x}, \mathbf{y}) \mid \mathbf{X}\lambda \leqslant \mathbf{x},\ \mathbf{Y}\lambda \geqslant \mathbf{y},\ \mathbf{1}_n\lambda = 1,\ \lambda_j \in \{0, 1\}; j \in J\}, \tag{1}$$

where $\mathbf{1}_n$ is a row vector with all components equal to one. Also, the other three FDH technologies, introduced by Kerstens and Vanden Eeckaut (1999), are as follows:

$$T^{\text{FDH-CRS}} = \{(\mathbf{x}, \mathbf{y}) \mid \mathbf{X}\lambda \leqslant \mathbf{x}, \mathbf{Y}\lambda \geqslant \mathbf{y}, \mathbf{1}_n\mathbf{w} = 1, \quad w_i \in \{0, 1\}, \lambda_i = \delta w_i, \delta \geqslant 0\}, \tag{2}$$

$$T^{\text{FDH-NIRS}} = \{(\mathbf{x}, \mathbf{y}) \mid (\mathbf{x}, \mathbf{y}) \in T^{\text{FDH-CRS}}, \delta \leqslant 1\}, \tag{3}$$

$$T^{\text{FDH-NDRS}} = \{(\mathbf{x}, \mathbf{y}) \mid (\mathbf{x}, \mathbf{y}) \in T^{\text{FDH-CRS}}, \delta \geqslant 1\}. \tag{4}$$

Modeling constant returns-to-scale (CRS), non-increasing returns-to-scale (NIRS), and non-decreasing returns-to-scale (NDRS) results from the above technologies, respectively.

The FDH input-oriented radial efficiency of DMU_o (x_o, y_o), in which $o \in J$, under different RTS assumptions of the reference technology is obtained by solving the following model:

$$\begin{aligned} \theta(x_o, y_o) = \min \quad & \theta \\ \text{s.t.} \quad & (\theta x_o, y_o) \in T, \end{aligned} \tag{5}$$

where T is defined by (1)–(4), upon the RTS assumption. DMU_o is considered as an (at least weak) efficient DMU, in each of the above models in (5), if $\theta(x_o, y_o) = 1$ in the respective model. It is obviously correct that

$$\theta_{\text{FDH-CRS}} \leqslant \theta_{\text{FDH-NIRS}}, \quad \theta_{\text{FDH-CRS}} \leqslant \theta_{\text{FDH-NDRS}}. \tag{6}$$

Using the K&VE method, the classification of RTS is obtained by comparing the $\theta_{\text{FDH-CRS}}$, $\theta_{\text{FDH-NIRS}}$, and $\theta_{\text{FDH-NDRS}}$. In this method if $(\mathbf{x}_o, \mathbf{y}_o)$ is an FDH-efficient point then:

(i) CRS $\Longleftrightarrow \theta_{\text{FDH-CRS}} = \max\{\theta_{\text{FDH-CRS}}, \theta_{\text{FDH-NIRS}}, \theta_{\text{FDH-NDRS}}\}$,
(ii) IRS $\Longleftrightarrow \theta_{\text{FDH-NDRS}} = \text{strict}\max\{\theta_{\text{FDH-CRS}}, \theta_{\text{FDH-NIRS}}, \theta_{\text{FDH-NDRS}}\}$,
(iii) DRS $\Longleftrightarrow \theta_{\text{FDH-NIRS}} = \text{strict}\max\{\theta_{\text{FDH-CRS}}, \theta_{\text{FDH-NIRS}}, \theta_{\text{FDH-NDRS}}\}$.

Note 1. The "strict max" expression is defined as: $a = \text{strict}\ \max\{a, b, c\}$ if and only if $a > b$ and $a > c$. This expression is not used in Kerstens and Vanden Eeckaut (1999), but we use this expression in order for the three conditions of the method to be disjoint and for further accuracy.