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Interfaces with Other Disciplines

Interval efficiency measures in data envelopment analysis with imprecise data

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Abstract

The conventional data envelopment analysis (DEA) measures the relative efficiencies of a set of decision making units (DMUs) with exact values of inputs and outputs. For imprecise data, i.e., mixtures of interval data and ordinal data, some methods have been developed to calculate the upper bound of the efficiency scores. This paper constructs a pair of two-level mathematical programming models, whose objective values represent the lower bound and upper bound of the efficiency scores, respectively. Based on the concept of productive efficiency and the application of a variable substitution technique, the pair of two-level nonlinear programs is transformed to a pair of ordinary one-level linear programs. Solving the associated pairs of linear programs produces the efficiency intervals of all DMUs. An illustrative example verifies the idea of this paper. A real case is also provided to give some interpretation of the interval efficiency. Interval efficiency not only describes the real situation in better detail; psychologically, it also eases the tension of the DMUs being evaluated as well as the persons conducting the evaluation.

Keywords: Data envelopment analysis; Efficiency; Imprecise data; Interval data; Ordinal data; Two-level mathematical programming

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1. Introduction

Data envelopment analysis (DEA) is a methodology for measuring the relative efficiencies of a set of decision making units (DMUs) that use multiple inputs to produce multiple outputs. Due to its solid underlying theoretical basis and wide applications to real world problems, much effort has been devoted to the research and development of this area since the pioneer work of Charnes et al. (1978). In the conventional

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DEA, all the data assume the form of specific numerical values. Cook et al. (1993, 1996) extend the data type to ordinal, where the data only have ordinal relations, without exact values. The faculty positions, e.g., full professor, associate professor, assistant professor, etc., and the academic degrees such as doctoral, master, and baccalaureate are typical ordinal data. Cooper et al. (1999, 2001) further discuss the case of interval data. The mixtures of interval data and ordinal data are referred to as *imprecise data* and the associated methodology as IDEA (imprecise DEA) by Cooper et al. (1999).

Intuitively, when data are imprecise, the efficiency scores calculated from the data should be imprecise as well. In other words, the efficiency scores should appear in ranges, rather than being exact values. When imprecision is taken into consideration, the associated DEA model becomes nonlinear, which makes its solution procedure difficult. Cooper et al. (1999) apply a scale transformation technique accompanied with variable alterations and successfully transform the nonlinear model to a linear one. Zhu (2003) simplifies their approach to reduce the computational burden. It will be shown in this paper that the efficiency scores calculated from their method are the upper bound of the efficiency intervals. Despotis and Smirlis (2002) develop a method to find the efficiency intervals for interval data. However, how to calculate the efficiency interval for ordinal data is not explicitly presented.

Hougaard (1999) proposes an idea of combining the efficiency calculated from the conventional DEA model with that of subjectively evaluated by experts to form fuzzy efficiency intervals. Notably, the observations are of exact value. Entani et al. (2002) consider the efficiency measure from a different point of view. Their idea is that the conventional DEA efficiency is the one measured under the most favorable (i.e., optimistic) condition, there should be another one measured under the least favorable (i.e., pessimistic) condition, and the efficiency of a DMU is an interval between the pessimistic and optimistic measures. The same idea can be applied to interval data as well. However, since the efficiency measure is an interval even for precise data, the efficiency interval constructed from interval data under this idea is very wide.

There are also approaches for measuring the efficiency for fuzzy data. Sengupta (1992) uses Zimmermann's method (1976) to calculate fuzzy efficiencies. Triantis and Girod (1998) apply the fuzzy linear programming approach of Carlsson and Korhonen (1986) to measure technical efficiency. Kao and Liu (2000) develop a solution procedure that is able to find the membership function of the fuzzy efficiency score. When the membership functions are rectangular, the fuzzy data boil down to interval data. León et al. (2003) use the possibilistic programming approach of Guo and Tanaka (2001) to find the efficiency scores at different possibilistic levels. In all these articles, ordinal data are not discussed.

Psychologically, interval efficiency measure is more favorable and acceptable to the DMUs being evaluated, especially in the Chinese societies. A DMU is reluctant to admit that it performs worse than another. An evaluator is also hesitated to announce that one DMU is performing less satisfactorily than another. The interval efficiency alleviates the perplexity brought by the harsh evaluation. Consider two DMUs A and B in two cases, one with exact efficiency scores of $E_A = 0.9$, $E_B = 1.0$ and the other with interval efficiency measures of $E_A = [0.8, 1.0]$, $E_B = [1.0, 1.0]$. In the former case, A is definitely worse than B because B is efficient while A is not. In the latter case, although A would probably not deny that it is worse than B because B is always efficient, it can claim that under some conditions it is also efficient.

In this paper, we apply a two-level mathematical programming approach to model the efficiency interval for imprecise data, including interval data as well as ordinal data. The two-level mathematical programming technique makes the modeling of the lower bound and upper bound of the efficiency interval very simple and clear. However, there are two problems need to be solved, one is two-level hierarchy and the other is nonlinearity. The next section introduces the formulation of the two-level mathematical programs. Then, we explain how the two-level nonlinear programs are transformed to the ordinary one-level linear programs. An example and a real case of imprecise data are exemplified to explain how the proposed methodology is applied to solving real world problems. More importantly, the physical meaning and the implication of the interval efficiency are interpreted.