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Continuous Optimization

A soft approach for hard continuous optimization

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Abstract

This paper is to introduce a soft approach for solving continuous optimizations models where seeking an optimal solution is theoretically or practically impossible.

We first review methods for solving continuous optimization models, and argue that only a few optimization models with some good structure are solved. To solve a larger class of optimization problems, we suggest a soft approach by softening the goal in solving a model, and propose a two-stage process for implementing the soft approach. Furthermore, we offer an algorithm for solving optimization models with a convex feasible set, and verify the validity of the soft approach with numerical experiments.

Keywords: Optimization; Optimal solution; Soft approach; Sampling

1. Introduction

Optimization is on how to choose an alternative so as to optimize certain objective function. Various optimization problems can be simply formulated by the following model:

$$\min \quad J(x): \ x \in X^f,$$

where $x \in R^n$ is a vector of decision variables, $J(x)$ is an objective function, and X^f is a set of feasible solutions, which is a huge or infinite set.

Solving an optimization model traditionally means seeking an optimal solution x^* in X^f. Beginning from 1940s, much research had been done on solving models this way, and many successful stories have been reported, such as the simplex method for linear programming, hill-climbing algorithms for nonlinear programming. Section 2 gives a brief overview on these methods, and argues that only models with some good analytical structures have been solved.

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When an optimization model is not of the good structures, how to solve the model has been a challenging problem. Section 3 introduces a new approach for optimization analysis, named a soft approach. After making clear the ideas in the soft approach, a two-stage process for implementing the soft approach is suggested. Section 4 offers an algorithm for solving continuous optimization models where the feasible set is a convex set. To illustrate the proposed soft approach, Section 5 applies the soft approach to solving two optimization problems. Section 6 concludes this paper and presents issues for further study.

2. Overview of methods for solving continuous models

Let the feasible set in continuous optimization problems be

$$X^f = \left\{ x \in \prod_{i=1}^{n} X_i | g_i(x) \leqslant 0, \quad i = 1, \ldots, m \right\}, \tag{1}$$

where $X_i = [a_i, b_i]$ is the domain of x_i, $g_i(x)$ is a constraint function ($i = 1,\ldots,m$).

The traditional linear programming, nonlinear programming, and the recent global optimization are the three branches on solving continuous optimization problems.

Linear programming is an optimization model where the objective function and all constraint functions in the model are linear.

The simplex method proposed by Danzig may be the most successful story in optimization, see the classic book by Danzig (1963) for more about LP. No more explanation on LP is necessary here, it is believed that LP is a solved model.

Nonlinear programming is an optimization model where one or more nonlinear functions are included in the model.

Many search methods, such as the Newton's method and the conjugate gradient method, had been proposed for seeking an optimal solution. You can find a lot of examples where these algorithms work well, but their limitations are also obvious. First, they require all functions in the model to be differentiable. Second, they require the objective function to be convex, otherwise, they only lead to stationary points or locally optimal solutions. Seeking an optimal solution of a nonconvex or/and nondifferentiable model has been a difficult problem. See Lasdon (1970), Luenberger (1984) and Fletcher (1987) for details about nonlinear programming.

Global optimization is a relatively new field which aims at seeking an optimal solution in nonlinear optimization models.

The main stream in global optimization exploits analytical properties of the model to generate a deterministic sequence of points converging to a global optimal solution. Two analytical properties, convexity and monotonicity, have been successfully exploited, giving rise to two research trends in global optimization: d.c. optimization and monotonic optimization.

The d.c. optimization deals with models described by differences of convex functions, while monotonic optimization deals with models described by means of functions monotonically increasing or decreasing along rays, hence the proposed algorithms still depend on these analytical properties of models. Refer to Horst et al. (1995), Horst and Pardalos (1995), for more about global optimization.

Therefore, we can see that only models with some good structures have been solved. For the general nonlinear model, how to find out an optimal solution remains to be a challenging problem.

Remark 1. Other methods for solving optimization problems have been proposed that do not necessarily aim at seeking an optimal solution with certainty, such as the stochastic approaches and the metaheuristics.

Stochastic approaches generally require little or no additional assumptions on the structure of optimization models, but the outcome they lead to is random, although some provide a probabilistic convergence guarantee, refer to Pardalos et al. (2000) for a review of three kinds of stochastic methods. These approaches are not practically usable since they can not quantify what a result they will produce within a given time period.

Metaheuristics are methods that are often based on processes observed in physics or biology. Three main categories of them are: simulated annealing,