European Journal of Operational Research 197 (2009) 1019–1027

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Optimization of capacity expansion planning for gas transportation networks

Jean Andre [a,*], Frédéric Bonnans [b], Laurent Cornibert [c]

[a] R&D Division, Gaz de France, 361 Av. du Pdt Wilson, 93211 St Denis, France
[b] INRIA-Saclay and Centre de Mathématiques Appliquées, Ecole Polytechnique, 91128 Palaiseau, France
[c] GRTgaz, Courcellor 1-2 Rue Curnonsky, 75017 Paris, France

ARTICLE INFO

Article history:
Received 28 November 2006
Accepted 11 December 2007
Available online 13 March 2008

Keywords:
OR in energy
Global optimization
Branch & Bound
Gas networks
Dimensioning

ABSTRACT

This paper presents techniques for solving the problem of minimizing investment costs on an existing gas transportation network. The goal of this program is to find, first, the optimal location of pipeline segments to be reinforced and, second, the optimal sizes (among a discrete commercial list of diameters) under the constraint of satisfaction of demands with high enough pressure for all users.

The paper develops new heuristics for solving this large-scale integer NLP problem, based on a two phases approach. The first one solves a continuous relaxation of the problem. A generalized potential formulation of the gas transportation networks including valves and compressor stations is introduced in order to find an initial point of the optimization solver. Phase two consists in choosing discrete values of diameters only among the set of pipes that have been reinforced in the continuous relaxation. A Branch & Bound scheme is then applied to a limited number of values in order to generate good solutions with reasonable computational effort on real-world applications.

1. Introduction

The French high-pressure natural gas transmission system consists of 31,589 km of pipelines (between 80 and 1000 mm in diameter). regional networks embedded in this system (with about 23,000 km) are high-pressure networks (between 20 and 68 bar) that bring gas from the main network (using compressor stations) to city distribution networks. These regional networks irrigate a limited geographical area and feed a substantial number of customers. These complex structures are made up of several pipe sections of different diameters. Each section has to be adapted to the various conditions of flow and pressure.

Although most of them are gunbarrel-like or tree-like systems, the largest networks have several supply nodes and contain numerous cycles (in the sense of graph theory).

In order to cope with increasing forecasted demand in gas, gas transportation companies need to plan the reinforcement of these regional transportation networks. The only way to increase capacity of these networks without compressor stations is to "loop" existing pipeline sections. In the natural gas industry, looping means that one pipeline is laid parallel to another. This is an important issue whose aim is to determine the cheapest set of diameters to be added each year, insuring that user's demands will be satisfied with high enough pressure.

Our paper focuses on the specific case when:

- A unique maximal scenario of demand is given (one stage problem).
- Regional transportation networks include only pipes and regulators (but no compressor stations).

As the diameters are to be chosen among a finite set of available values on each arc, this optimization problem is highly combinatorial.

The problem of designing a piping system has been widely studied in the literature. To tackle this problem in reasonable computation times, a first class of papers dealing with pipe-network design (of either water or gas) uses meta-heuristics such as genetic algorithms, see (Abebe and Solomatine, 1998; Boyd et al., 1994; Surry et al., 1995; Van Vuuren, 2002). A second class of papers uses methods based on continuous relaxation. Hansen et al. (1991) use a trust-region successive linear programming method. Their algorithm directly handles the discrete choice of diameter but each step (in which the variation of diameter is continuous) needs a linearization of the objective function and constraints, as well as a procedure for adjusting the diameter in order to satisfy the lower bound on pressures. De Wolf and Smeers (1996) deal only with the continuous variables of diameter. Their objective function combines the cost of purchasing gas at supply nodes and the investment cost on the network. They solve the resulting nonsmooth optimization problem using a bundle algorithm. Zhang and Zhu (1996) propose to model the combinatorial aspect with one binary variable per diameter on

* Corresponding author.
E-mail addresses: jean-dr.andre@gazdefrance.com (J. Andre), Frederic.Bonnans@inria.fr (F. Bonnans), laurent.cornibert@grtgaz.com (L. Cornibert).

0377-2217/$ - see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.ejor.2007.12.045

Nomenclature

N	set of nodes of the network
A:	set of arcs (union of sets A_{pipe} and A_{reg})
A_{pipe}:	set of pipes
A_{reg}:	set of regulators
P^i, $\pi^i := (P^i)^2$	pressure and energy head associated with node i
Q_a, DI_a, DD_a	flow, initial diameter and doubling diameter on arc a (since the network is expanded by laying out new pipes in parallel what is called looping)
M	the node-arc incidence matrix, whose (column) partition for pipes and regulators are called M_{pipe} and M_{reg}, respectively. Inside these sparse matrices, the only nonzeros elements give an arbitrary direction to the arc a by setting 1 for the inlet node and -1 for the outlet node.

each arc associated with a choice constraint on these variables (with a sum of binary variables equal to one). As they consider the continuous relaxation of their binary variables, they assume that they can split an arc into several parts, each one associated with only one discrete value of diameter. They prove that it is not optimal to split an arc into more than two parts. In order to compute a solution, they reformulate the problem as a bilevel program and use trust-region methods. Let us remind the paper by Osiadacz and Gorecki (1995) where a sequential quadratic algorithm is applied to a continuous relaxation. Flowrate variables are eliminated by assuming the gas speed on each pipeline to be constant. The continuous solution is then rounded to the nearest discrete diameter.

This paper clearly belongs to the second class of papers but differs in two ways from the above references. First, we consider the reinforcement problem for already existing networks. This increases the nonconvexity of the model and the size of the problem. Second, we introduce a combination of continuous relaxation and B&B algorithm.

The paper is organized as follows: Section 2 briefly states the problem to be solved. The continuous relaxed problem, described in Section 3, provides the starting point of the Branch & Bound algorithm presented in Section 4.

Numerical results on real regional networks are displayed in Section 5. We conclude the paper in Section 6.

2. Framework

In order to transport gas on large regional networks, high-pressures are used at entry points. The regional networks considered in this study include two types of elements: on the one hand, pipelines along which pressure drops depend upon diameters and flows and, on the other hand, regulators. The latter allow additional pressure drops in order to comply with constraints on the maximum admissible operational pressure of downstream networks.

Expanding capacities of regional networks means to identify which (existing) pipe sections to reinforce, and to lay new pipelines along these existing sections (such a process is called "looping").

Our approach proceeds in two steps: (a) determination of the location of the pipelines to be looped, and (b) choice of the size of the new parallel pipelines. With every doubling diameter DD_a on arc a is associated a cost given by the stepwise function $c_a(\text{DD}_a)$ depending on the laying cost, the steel price, and the length of the pipe section.

Since we wish to minimize the investment cost, i.e., the sum of reinforcement costs, the resulting model is stated as follows:

$$\begin{cases} \min\limits_{(\text{DD},Q,\pi)} \sum\limits_{a\in A_{\text{pipe}}} c_a(\text{DD}_a) \\ \text{(i)} \quad \text{DD}_a \in \{0, \Delta_a^1, \ldots, \Delta_a^k, \ldots, \overline{\Delta_a}\} \quad \text{for all } a \in A_{\text{pipe}} \\ \text{(ii)} \quad \pi_a^i - \pi_a^j = C_a(\text{DI}_a^s + \text{DD}_a^s)^{-5/s} Q_a|Q_a| \quad \text{for all } a \in A_{\text{pipe}} \\ \text{(iii)} \quad \underline{\pi} \leqslant \pi \leqslant \overline{\pi} \\ \text{(iv)} \quad M_{\text{reg}}^T \pi \geqslant 0 \\ \text{(v)} \quad 0 \leqslant \underline{Q_a} \leqslant Q_a \leqslant \overline{Q_a} \quad \text{for all } a \in A_{\text{reg}} \\ \text{(vi)} \quad MQ = b \end{cases} \tag{1}$$

with $s := 5/2$ and $C_a > 0$ given constant depending on the length of arc a.

Constraint (i) imposes that the diameter value, for each pipe a, is to be chosen among a finite set $\{\Delta_a^k\}$ including 0, the largest element of which is denoted by $\overline{\Delta_a}$. Constraint (ii) represents the Weymouth pressure drop equation on pipe (Katz et al., 1959). Constraint (iii) requires to keep the pressures within minimal bounds $\underline{\Pi}$ (delivery pressures) and maximal bounds $\overline{\Pi}$ (maximum admissible operational pressures, supply pressures). Constraint (iv) ensures the fall of pressure on regulators. Constraint (v) controls flow rates on regulators within minimal and maximal bounds $\underline{Q}$ and $\overline{Q}$. Constraint (vi) guarantees Kirchhoff's law of flow conservation.

The nonconvexity of program (1) is due to the discrete nature of diameters and associated cost function, and to the nonlinearity of the pressure loss equations.

In order to move a part of the nonlinearity from the constraints to the objective function, a first change of variables has been made: instead of doubling diameters, we use "equivalent diameters", whose expression is[1] $\text{Deq}_a := (\text{DI}_a^s + \text{DD}_a^s)^{1/s}$. Program (1) can be rewritten as follows:

$$\begin{cases} \min\limits_{(\text{Deq},Q,\pi)} \sum\limits_{a\in A_{\text{pipe}}} \tau_a(\text{Deq}_a) \\ \text{(i)} \quad \text{Deq}_a \in \{\text{DI}_a, \text{Deq}_a^1, \ldots, \text{Deq}_a^k, \ldots, \overline{\text{Deq}_a}\} \quad \text{for all } a \in A_{\text{pipe}} \\ \text{(ii)} \quad \pi_a^i - \pi_a^j = C_a \text{Deq}_a^{-5} Q_a|Q_a| \quad \text{for all } a \in A_{\text{pipe}} \\ \text{(1-iii) to (1-vi)} \end{cases} \tag{2}$$

with $\tau_a(\text{Deq}_a) := c_a((\text{Deq}_a^s - \text{DI}_a^s)^{1/s})$ and $\overline{\text{Deq}_a}$, the maximal equivalent diameter corresponding to the maximal doubling diameter $\overline{\Delta_a}$. The resulting program is (again) a mixed nonlinear, nonconvex program, having discrete variables Deq and continuous variables π and Q.

3. Phase I: continuous relaxation

This section is dedicated to the formulation and the resolution of a continuous relaxation of the problem, for which a feasible point can be computed by solving a convex problem (the potential formulation of the network equations).

3.1. Relaxed program

The continuous relaxation allows the values Deq_a of diameters of additional pipes to be selected within a certain interval:

$$\text{Deq}_a \in [\text{DI}_a, \overline{\text{Deq}_a}] \quad \text{for all } a \in A_{\text{pipe}}$$

We extend function τ_a to $[\text{DI}_a, \overline{\text{Deq}_a}]$, by making it equal to $\tau_a(\text{Deq}_a^{k+1})$ over $[\text{Deq}_a^k, \text{Deq}_a^{k+1}]$. This stepwise function can be approximated by a continuous concave function:

$$\phi_a(\text{Deq}_a) = \alpha_a(\text{Deq}_a^s - \text{DI}_a^s)^{1/s}$$

[1] With two pipes in parallel (1&2) and the gas flowing in the same direction for both pipes, we can write $Q_a = ((\pi_a^i - \pi_a^j) \cdot C_a^{-1} \cdot D_a^5)^{1/2}$, $a = 1, 2$. Substituting these expressions of flows in the following expression: $\text{Deq} = (C_a.(\pi_a^i - \pi_a^j)^{-1}.(Q_1 + Q_2)^2)^{1/5}$, we obtain the equivalent diameter formula.