Available online at www.sciencedirect.com

SCIENCE DIRECT®

European Journal of Operational Research 172 (2006) 798–813

EUROPEAN JOURNAL OF OPERATIONAL RESEARCH

www.elsevier.com/locate/ejor

Discrete Optimization

A recovering beam search algorithm for the single machine Just-in-Time scheduling problem

B. Esteve, C. Aubijoux, A. Chartier, V. T'kindt *

Laboratoire d'Informatique (EA2101), Université François-Rabelais de Tours, 64 Avenue Jean Portalis, 37200 Tours, France

Received 31 March 2003; accepted 10 November 2004
Available online 19 January 2005

Abstract

We consider the Just-in-Time scheduling problem where the Just-in-Time notion is captured by means of multiple conflicting criteria. The calculation of any non-dominated solution for these criteria is achieved by solving an extension of the single machine problem of minimising the mean weighted deviation from distinct due dates. In the extended problem each job to schedule is also constrained by a release date and a deadline. This problem is $\mathcal{NP}$-Hard in the strong sense and we propose heuristic algorithms to solve it. Computational experiments show that, among those algorithms, the most effective heuristic, in terms of quality, is a Recovering Beam Search algorithm.

Keywords: Just-in-Time scheduling; Multicriteria optimisation; Recovering beam search

1. Introduction

In this paper the single machine Just-in-Time (JIT) scheduling problem is considered. It can be stated as follows. A set of n jobs must be processed on a single machine which is continuously available. Each job j has a processing time p_j and a due date d_j and incurs a unitary earliness penalty α_j and a unitary tardiness penalty β_j. The machine processes one job at a time and can be left idle if necessary. Preemption is not allowed. Given any schedule we denote by C_j the completion time of job j. We make use of E_j and T_j to refer to the earliness and tardiness of job j, respectively, and we have $E_j = \max(0; d_j - C_j)$ and $T_j = \max(0; C_j - d_j)$. Formally we define the JIT problem as a multicriteria problem by introducing n criteria Z_j, $\forall j = 1, \ldots, n$, where $Z_j = \alpha_j E_j + \beta_j T_j$ is the cost generated by job j, namely its weighted deviation.

* Corresponding author. Tel.: +33 2 47 36 14 14; fax: +33 2 47 36 14 22.
E-mail addresses: bertrand.esteve@etu.univ-tours.fr (B. Esteve), tkindt@univ-tours.fr (V. T'kindt).

0377-2217/$ - see front matter
doi:10.1016/j.ejor.2004.11.014

The objective is to find strictly non dominated schedules or strict Pareto optimal schedules for these criteria. A schedule s is a strict Pareto optimum if and only if there does not exist another schedule s' such that $Z_j(s') \leqslant Z_j(s)$, $\forall j = 1, \ldots, n$, with at least one strict inequality.

This problem is called the single machine Just-in-Time scheduling problem since it is closely related to the principles of this operations management philosophy, indeed meeting customer's demand as close as possible to his requirements (i.e. delays, as far as scheduling is concerned) with no waste (i.e. at a minimum cost). In scheduling, this situation is modeled by tardiness penalties that represent the customer dissatisfaction if products are delivered late and by earliness penalties that represent storage costs for finished goods or deterioration costs for perishable goods, maybe occurring if the product is completed early. The proposed multicriteria problem enables a decision maker to explicitely choose the trade-off between jobs, i.e. he can prefer a strict Pareto optimum in which a job j is on-time and a job k is early whilst another strict Pareto optimum exists in which the opposite situation occurs. Thus, considering multiple criteria to capture the Just-in-Time aspect of the problem, offers more flexibility than in classic JIT scheduling problems. It is the case for the early/tardy single machine scheduling problem with distinct due dates which, using the classic scheduling notation [11], can be denoted by $1|d_j|\sum \alpha_j E_j + \beta_j T_j$. Notice that in this problem, a linear combination of criteria Z_j is minimised. Consider the following 3-job example.

j	p_j	d_j	α_j	β_j
1	5	15	2	2
2	3	19	4	1
3	4	22	1	3

Six sequences can be considered, each one yielding several schedules. For each sequence the jobs are scheduled continuously and the different schedules are obtained by changing the start time of the first job of the sequence. We have 62 interesting schedules, among which 3 are strict Pareto optima. The following table shows the earliness and tardiness penalties for each job as well as the values of criteria Z_j and the value of the objective function $\sum \alpha_j E_j + \beta_j T_j$:

Sequence	$\alpha_1 E_1$	$\beta_1 T_1$	Z_1	$\alpha_2 E_2$	$\beta_2 T_2$	Z_2	$\alpha_3 E_3$	$\beta_3 T_3$	Z_3	$\sum \alpha_j E_j + \beta_j T_j$
(1;2;3)	0	0	0	4	0	4	0	0	0	4
	0	2	2	0	0	0	0	3	3	5
(1;3;2)	0	0	0	0	3	3	3	0	3	6

For the classic $\sum \alpha_j E_j + \beta_j T_j$ objective function there is one optimal solution with a value of 4. However, from the point of view of criteria Z_j, the situation is not so simple because the optimal solution for the above objective function has an evaluation vector of $(Z_1; Z_2; Z_3) = (0; 4; 0)$ where the costs are achieved by scheduling job 1 and job 3 on time and job 2 early. But there is also another solution, with an evaluation vector of $(Z_1; Z_2; Z_3) = (2; 0; 3)$ and an objective function value of 5. The evaluation vector is quite the same but the costs are generated by scheduling job 1 late and job 3 late. Henceforth, the decision maker may prefer this latter solution which minimizes the maximum of Z_1, Z_2, Z_3, if having a job with a cost greater then 4 is not acceptable. This example shows that considering criteria Z_j provides more flexibility than a classic $\sum \alpha_j E_j + \beta_j T_j$ objective function when choosing a JIT schedule.

In this paper we are interested in the calculation of strict Pareto optima for criteria Z_j. This is achieved by means of the parametric analysis introduced in the following theorem.

Theorem 1 [24]. *Let S be the set of solutions, Z the set of corresponding criteria vectors and g a strictly increasing function from $\mathbb{R}^n$ to $\mathbb{R}$ and lower bounded on Z. $x^0 \in S$ is a strict Pareto optimum if and only if $\exists b \in \mathbb{R}^n$ such that x^0 is an optimal solution of the following problem* ($P_{(g,b)}$):