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Continuous Optimization

ISTMO: An interval reference point-based method for stochastic multiobjective programming problems

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ABSTRACT

In this paper, we present an interactive algorithm (ISTMO) for stochastic multiobjective problems with continuous random variables. This method combines the concept of probability efficiency for stochastic problems with the reference point philosophy for deterministic multiobjective problems. The decision maker expresses her/his references by dividing the variation range of each objective into intervals, and by setting the desired probability for each objective to achieve values belonging to each interval. These intervals may also be redefined during the process. This interactive procedure helps the decision maker to understand the stochastic nature of the problem, to discover the risk level (s)he is willing to assume for each objective, and to learn about the trade-offs among the objectives.

1. Introduction

When facing the analysis and resolution of a real decision problem using a mathematical optimization model, we can find situations where the values of some important elements (parameters) for the decision making process are unknown at the moment when the decision is made. Solving such a problem is not a trivial task, because an objective function must be optimized, but we do not know the values of this function because they depend on random parameters. The fact that the achievable objective values are uncertain makes the election harder, because a random variable does not allow an order relation. One typical problem with these features is portfolio optimization. The election of the securities for the portfolio is not a typical optimization problem, because the future evolution of the prices of the securities is unknown. In these cases, the decision is said to be risky.

The previous comments might lead us to conclude that, when an optimization problem has a stochastic objective function (that is, the function depends not only on the decision variables, but also on random parameters), each of the possible elections may be equally "good" or "bad". But this is not certain in general. Even though we cannot know the value that the objective function will achieve (in the previous example, the future prices of the securities), if we know the probability distribution of the stochastic objective, it is possible to obtain a "partial order", using the statistical features (expected value, variance, quantiles, etc.). Basically, in such a problem, two criteria must be taken into account so as to obtain this partial order: which level can be regarded as satisfactory for the objective function, and how much risk do we want to assume.

The solution processes that can be found in the scientific literature make use in some way of these ideas. In most of them, the two aforementioned criteria, expected value and risk, are taken into account. The differences between the solution methods lay on the importance given to these two criteria and on the way the decision maker's preferences are considered. Among all the criteria used in the literature, the most widely used in real problems is the efficiency criterion called expected value-variance, proposed by Nobel Prize Markowitz (1952). When this criterion is applied, the stochastic problem is transformed into a deterministic one with two objectives, where the expected value of the stochastic objective is to be maximized. Besides, the objective's variance is used as a risk measure (so that it is understood that if the variance is minimized, so is the risk associated to the objective). Making use of this idea, a set of efficient expected value-variance solutions is generated. This set is shown to the decision maker, who has to choose the solution which best fits his/her preferences.

On the other hand, real decision problems always involve the simultaneous consideration of several conflicting criteria. These facts motivate the study of the so-called stochastic multiobjective programming (SMOP) problems.

The solution of this kind of problem is far from being a trivial task, because two of the main hypotheses of classical mathematical programming are relaxed: the values of some of the parameters of the problem are unknown, and the decision maker (DM) wishes to optimize several conflicting criteria at the same time. Many theoretical works that tackle these problems can be found in the scientific literature. Among them, the books of Goicoechea et al. (1982),

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Slowinski and Teghem (1990), Stancu-Minasian (1984) can be highlighted, where the issue of determining solutions for SMOP is analyzed from different points of view: efficiency analysis, determination of satisfying solutions, compromise programming approaches, use of interactive procedures, etc. On the other hand, some applications of SMOP problems to real decisions have also been reported in the literature. For example, in Ben Abdelaziz and Mejri (2001), a stochastic multiobjective model is proposed to determine the appropriate releases decisions for various water reservoirs to satisfy multiple conflicting objectives (minimization of salinity, minimization of pumping costs). In Ballestero (2001), a stochastic problem is used to manage a farm and in Ballestero (2005) nine real stochastic goal programming applications are described. In both works, the author claims that the solutions of some economic and technological problems, which are usually treated as deterministic models, can be improved if the models are reformulated as stochastic ones.

In order to solve a multiobjective problem, in such a complex environment as in SMOP, the DM needs to rely on a method that can assist him/her in learning about the real structure of the problem. Besides, the method must also be capable of guiding him/her through the, usually very large, efficient set, towards his/her most preferred solution. Interactive methods have proved to be the most adequate ones for this double task: learning and guiding. The basic structure of an interactive method is very simple. One (or several) efficient solution is determined and shown to the DM. If (s)he is satisfied with the current solution, the procedure ends. Otherwise, the DM is required to give additional information about her/his preferences; next, a new efficient solution is determined using this information, and so on (see Miettinen (1999) for a description and overview of interactive procedures).

Not that many interactive methods for SMOP problems have been reported in the literature. Besides, most of them have been specifically designed for SMOP problems with discrete random variables. Namely, the PROMISE (Urli and Nadeau, 1990), STRANGE (Teghem et al., 1986) and PROMISE/scenarios (Urli and Nadeau, 2004) methods are meant to solve problems with discrete random parameters or with incomplete information. Specifically, the PROMISE method solves problems where the probability distributions of the random parameters are unknown, and only incomplete information about them (worst value, best value and central tendency measure) is available. The STRANGE, and PROMISE/scenarios methods have been developed to solve problems where the random parameters are represented by discrete random variables and take a finite number of values that are assigned known subjective probabilities (scenarios). All these procedures extend the well-known deterministic interactive method STEM (see Benayoun et al. (1971)) to SMOP problems. These interactive methods have the same common feature: in the interactive phase, if the DM is not satisfied with the current solution, (s)he is requested to choose a single objective, which is the objective that the DM wishes to improve in the next iteration (STRANGE), or the objective that the DM allows to become worse in order to improve others (PROMISE and PROMISE/scenarios) must be chosen. This fact can involve placing the rest of the objectives in a second level. We believe that it is better to let the DM express her/his opinions about all the objectives, so that the next iteration will better reflect her/his preferences and trade-offs among the criteria.

On the other hand, the only method designed so far for SMOP problems where the random parameters are continuous random variables with known probability distributions (which is the problem faced in this paper) is the PROTRADE method (Goicoechea et al. (1982)) This method is designed for stochastic problems with non-linear objective functions and constraints and establishes general hypotheses about these probability distributions. The main features of this method are the following:

- At each iteration, a linear combination of the expected values of the stochastic objective functions is optimized. Therefore, the solutions obtained are weakly efficient with respect to the expected value criterion (or properly efficient if all the weights of the linear combination are strictly positive). In Section 3.1, it will be seen that this efficiency criterion is outperformed in many cases by others.
- In order to obtain the weights, a multiattribute utility function is used to interact with the DM. This is an important drawback, because the elicitation of the parameters of a hypothetical utility function is, in general, a very hard task (if not impossible) for the DM.
- In the interactive phase if the DM is not satisfied with the current solution, (s)he is requested to choose a single objective, which is the objective that the DM wishes to improve in the next iteration (like in STRANGE). For this objective function, the DM must fix a level to be reached, and a probability. These preferences are incorporated to the optimization as a hard probabilistic constraint, and this modifies the feasible set of the problem, which may be empty at some point. Besides, this reduces the number of possible iterations to the number of objectives of the problem. If the DM is not satisfied with the solution after these iterations, the authors propose to re-estimate the parameters of the utility function. Therefore, the process cannot be regarded as comfortable and intuitive for the DM.
- The information shown to the DM at each step is the expected value of each objective function, and the probability to achieve this value. But although the expected value is a significant piece of information, it does not provide by itself an overall view of the possible values that the stochastic objective can take.

In our opinion, there are three key features for the success of an interactive stochastic method. First, the DM must find it reasonably comfortable and natural to give the information required by the method. This is important because otherwise the DM will very likely give inconsistent information, and consequently, the procedure will not be reliable. Besides this, the DM also needs to rely on the procedure, so that (s)he can learn about the structure of the problem throughout the process. Second, based on this reliable information, the algorithm must be able to find (or at least, to approximate as closely as possible) the DM's most preferred solution in a reasonable number of iterations. Finally, the method must aid the DM in understanding the stochastic nature of the problem, showing her/him the risk level associated with each solution, and thus, helping her/him to determine the risk (s)he is prepared to assume. These three aspects have been taken into account to develop the interactive procedure described in this paper.

Taking into account all these considerations, we have developed a new interactive method, called ISTMO (Interval STochastic MultiObjective programming algorithm), based on the achievement scalarizing function approach, which adapts the reference point philosophy to stochastic problems. To this end, we propose a novel modification of the classical achievement functions, which allows the consideration of probabilities given by the DM. Therefore, the idea underlying this algorithm is to make use of the benefits of the reference point approach at the same time that the probabilistic aspects of the problem are incorporated in the iterations. This interactive method can be used to solve both single objective and multiobjective problems, with continuous random parameters. The main idea of the algorithm is to let the DM divide the range of possible values of each objective into some intervals (e.g. intervals of values regarded as "very poor", "poor", "fair", "good" and "very good"), and to let her/him control, during the interactive process, the probability for the objective to get values inside each