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Joint statistical design of double sampling $\overline{X}$ and s charts

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Abstract

In statistical quality control, usually the mean and variance of a manufacturing process are monitored jointly by two statistical control charts, e.g., a $\overline{X}$ chart and a R chart. Because of the efficiency of double sampling (DS) $\overline{X}$ charts in detecting shifts in process mean and DS s charts in process standard deviation it seems reasonable to investigate the joint DS $\overline{X}$ and s charts for statistical quality control. In this paper, a joint DS $\overline{X}$ and s chart scheme is proposed. The statistical design of the joint DS $\overline{X}$ and s charts is defined and formulated as an optimization problem and solved using a genetic algorithm. The performance of the joint DS $\overline{X}$ and s charts is also investigated. The results of the investigation indicate that the joint DS $\overline{X}$ and s charts offer a better statistical efficiency in terms of average run length (ARL) than combined EWMA and CUSUM schemes, omnibus EWMA scheme over certain shift ranges when all schemes are optimized to detect certain shifts. In comparison with the joint standard, two-stage samplings and variable sampling size $\overline{X}$ and R charts, the joint DS charts offer a better statistical efficiency for all ranges of the shifts.
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Keywords: Statistical quality control; Double sampling; Joint statistical design; Genetic algorithm

1. Introduction

In statistical quality control, usually the mean and variance of a manufacturing process are monitored jointly. Over the past years, most of the research on joint monitoring of process mean and process variance has focused on the joint $\overline{X}$ and R charts (see, for example, Jones and Case, 1981; Rahim, 1989; Saniga, 1991; Costa, 1993, 1998; Costa and Rahim, 2000).

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Particularly, Saniga (1991) presented a joint statistical design of $\overline{X}$ and R chart. A procedure coded in FORTRAN enables users to determine the control limit parameters and sample size of a jointly designed $\overline{X}$ and R chart for a specified statistical criterion. This criterion can be stated in terms of average run length (ARL), Type I and Type II error probabilities, or average time to signal (ATS).

Gan (1995) developed control schemes for joint monitoring of process mean and variance by using exponentially weighted moving average (EWMA) control charts. The first scheme, EE_U was obtained by running a two-sided EWMA mean chart and a high-side EWMA variance chart simultaneously. A two-sided mean chart (Crowder, 1987a,b, 1989; Lucas and Saccuci, 1990) is obtained by plotting sample statistic $Q_t = (1-\lambda_M)Q_{t-1} + \lambda_M \overline{X}_t$ against sample number t for $t=1, 2, \ldots$ A high-side variance chart (Crowder and Hamilton, 1992) is obtained by plotting sample statistic $Q_t = \max\{0, (1-\lambda_V)Q_{t-1} + \lambda_V \log(s_t^2)\}$ against sample number t for $t=1, 2, \ldots$, where s_t^2 is the sample variance. This is a special case of the generalized charting procedure introduced by Champ et al. (1991). The second scheme, EE consists of a two-sided EWMA mean chart and a two-sided EWMA variance chart. A two-sided EWMA variance chart is obtained by plotting $Q_0 = E\{\log(s_t^2)\} = -0.270$ (when process variance σ^2 is equal to target process variance σ_0^2) and $Q_t = \{(1-\lambda_V)Q_{t-1} + \lambda_V \log(s_t^2)\}$ against sample number t for $t=1, 2, \ldots$.

In Gan (1995), the EE_U and EE schemes were compared with a combined two-sided CUSUM (CC) scheme and a omnibus EWMA chart scheme (Domangue and Patch, 1991). The results of the comparison showed that the omnibus charts and EE_U scheme are ineffective in detecting out-of-control situations characterized by a shift in process mean with simultaneous decrease in variance. For adequate joint monitoring, it was suggested that a combined scheme like EE and CC should be used. The schemes like EE and CC are very similar in performance and they are often more sensitive than the omnibus charts in detecting other out-of-control situations. The omnibus charts have difficulties in meaningful interpretations of the out-of-control signals when an out-of-control signal is issued since an omnibus chart does not indicate whether the signal is due to the mean shift or the variance shift.

The CC scheme is obtained by plotting sample statistics $S_t = \max\{0, S_{t-1} + \overline{X}_t - k_M\}$ and $T_t = \min\{0, T_{t-1} + \overline{X}_t + k_M\}$ against sample number t for the mean chart and by plotting $V_t = \max\{0, V_{t-1} + \log(s_t^2) - k_{VU}\}$ and $K_t = \min\{0, K_{t-1} + \log(s_t^2) - k_{VL}\}$ against sample number t for the variance chart.

The omnibus EWMA scheme proposed by Domangue and Patch (1991) is based on statistic $A_i = \lambda|Z_i|^\alpha + (1-\lambda)A_{i-1}$ for $i=1, 2, \ldots$, where $0<\lambda \leqslant 1$. The authors chose the EWMA of $|Z|^\alpha$ for some α because the proposed procedure was shown to be sensitive to changes in location when $\sigma \geqslant \sigma_0$ and to increases in dispersion. The authors mentioned that the proposed chart would not be effective for detection of a small change in mean when $\sigma<\sigma_0$. For simplicity they considered only cases with $\alpha=0.5$ and $\alpha=2$. A "signal" to stop or inspect the process is given whenever $A_i \geqslant E[A_i] + L\,\mathrm{var}[A_i]^{1/2}$. The quantity L is specified by the user. For $i\to\infty$, $E[A_i] = (\frac{2^\alpha}{\pi})^{1/2}\Gamma[(\alpha+1)/2]$, and $\mathrm{var}[A_i] = \frac{2^\alpha\lambda}{(2-\lambda)\pi}[\sqrt{\pi}\Gamma[\alpha+0.5] - (\Gamma[(\alpha+1)/2])^2]$. The omnibus EWMA schemes with different design parameters were compared with other schemes, particularly CUSUM of $|Z|^\alpha$ proposed by Hawkins (1981) and Healy (1987), which is referred to as omnibus CUSUM procedure. This procedure involves keeping a single sum given by $Y_i = \max[0, |Z_i|^\alpha - k + Y_{i-1}]$. The process stops whenever control statistic Y_i exceeds h, which is referred to as the decision value.

The comparison results in Domangue and Patch (1991) show that for all possible scenarios the omnibus EWMA and CUSUM charts are better than the joint $\overline{X}$ and R chart, the individual moving range (IMR) charts, the EWMA for mean, the $\overline{X}$, and the $\overline{X}$ with warning limits.

Chen et al. (2001) proposed a new EWMA (MaxEWMA) chart which combines two sample statistics, i.e., mean and variance into one. Let U_i be the ith standardized sample mean and $V_i = \Phi^{-1}\{H(\frac{(n_i-1)s_i^2}{\sigma^2}; n_i - 1)\}$, where $\Phi(z)=P(Z \leqslant z)$ for $Z\sim N(0,1)$, the standard normal distribution, $\Phi^{-1}(\bullet)$ is the inverse function of $\Phi(\bullet)$, and $H(w,v)=P(W \leqslant w|v)$ for $W \sim \chi_v^2$. U_i and V_i are independent because $\overline{X}$ and s_i are independent. Since both U_i and V_i have the same distribution then a single chart could be constructed to monitor both the process mean and the process variability. First, they defined two EWMA charts $Y_i=(1-\lambda)Y_{i-1}+\lambda U_i$ and $Z_i=(1-\lambda)Z_{i-1}+\lambda V_i$, $i=1, 2, \ldots$. Then the two charts are combined

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