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On the Hungarian inventory control model

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Abstract

In this paper we recall and further develop an inventory model formulated by the author [Prékopa, A., 1965. Reliability equation for an inventory problem and its asymptotic solutions. In: Prékopa, A. (Ed.), Colloquia Applied Mathematics in Economics. Publ. House of the Hung. Acad. Sci., Budapest, pp. 317–327; Prékopa, A., 1973. Generalizations of the theorems of Smirnov with application to a reliability type inventory problem. Math. Operationsforschung und Stat. 4, 283–297] and Ziermann [Ziermann, M., 1964. Application of Smirnov's theorems for an inventory control problem. Publications of the Mathematical Institute of the Hungarian Academy of Sciences Ser. B 8, 509–518] that has had wide application in Hungary and elsewhere. The basic assumption made in connection with this model is that the delivery of the ordered amount takes place in an interval, according to some random process, rather than at one time epoch. The problem is to determine that minimum level of safety stock, that ensures continuous production, without disruption, by a prescribed high probability. The model is further developed first by its combination with another inventory control model, the order up to S model and then, by the formulations of a static and a dynamic type stochastic programming models.

Keywords: Inventory control; Stochastic programming; Dynamic programming

1. Introduction

It was 40 years ago when the president of the National Planning Board of Hungary, a mathematically educated engineer proposed the problem to the Mathematical Institute of the Hungarian Academy of Sciences, to explain and remedy the problem of superfluously high inventory levels at Hungarian industrial plants. The problem was formulated in such a way that 1% increase in production necessitates how much percent increase in the inventory levels? Supply department managers usually increased the inventory levels proportionally with production increase and this policy resulted in huge inventories nationwide.

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The Operations Research Department (called Applications of Mathematics in Economics at that time), headed by the author, followed up the problem. After a long lasting (almost one year) diagnostic activity, Prékopa and Ziermann, who had been working on the problem, found that most of the costs, used in existing inventory control models in the international literature, were unknown and the high levels of the superfluous inventories were the results of the uncertainties in the deliveries. The delivery that followed an order took place in an interval, rather than at one time epoch, at random times in random quantities. We call it random interval delivery.

Inventory control theory had already existed at the time we conducted our investigations. Since the pioneering paper by Arrow et al. (1951) many papers appeared in that area and the publication of the comprehensive book by Hadley and Whitin (1963) was underway. Our problem, however, did not fit into the existing models and we had to look for solution elsewhere. Finally we realized that order statistics was the branch of science that could provide us at least with a starting point to describe the random delivery processes. That was the first major step. Later on some of the order statistical theorems have been generalized, and, by the use of them, very good solutions to the practical problems have been obtained.

The obtained models and methods, that allowed for the calculation of the safety stock in a simple way, became widely applied in Hungary, in a short time. Inventory levels of thousands of raw materials and semi-finished products had been set, by the use of the new formulas, and huge savings had been reported (the unofficial figure of the savings was four billion HUF in the middle of the 1960s which can be about the same amount of today's USD).

The applications of the model constructions worked out in connection with the random interval delivery problem went beyond the boundaries of Hungary (signals about them came from Czechoslovakia, Germany, etc.) but the models have not become as widely known as they deserve it. In fact, the problem seems to exist in many different contexts. For example Segal (1997) reported about its existence in the paper industry and Morris et al. (1987) in the utility industry.

The purpose of this paper is to call the attention to this model system and present some new variants of it. In Section 2 we describe the original Hungarian inventory control model. In Sections 3 and 4 we show that it can be combined with more traditional inventory control models. We take the "order up to S model" as an example. In Section 5 more general stochastic programming formulation of the original model is presented. Finally, in the last Section 6 we present a dynamic type stochastic programming model for the solution of inventory control problems with interval type deliveries and safety constraints.

2. The Hungarian inventory control model

Consider a finite time interval $[0, T]$. Assume that prior to time 0 Company A and B agree that Company A will deliver a given amount of some raw material or semi-finished product (briefly material) to Company B, that the latter will use (consume) for production of some product(s). There is no agreement, however, regarding the scheduling of the delivery. The amount of material delivered in $[0, T]$ is supposed to be equal to the amount Company B consumes in the same time interval. The problem is to find that minimal safety stock level M that ensures continuous production (consumption), without disruption, by probability $1 - \epsilon$, where ϵ is a given small number.

2.1. The basic model

In connection with the delivery and consumption processes we assume the following:

(a) Deliveries take place at discrete times, the number of which is fixed and is equal to n, that we can estimate from past history.The n delivery times are random, and their joint probability distribution is the