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Stochastics and Statistics

Economic and economic-statistical design of a chi-square chart for CBM

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Abstract

In this paper, the economic and economic-statistical design of a χ^2 chart for a maintenance application is considered. The machine deterioration process is described by a three-state continuous time Markov chain. The machine state is unobservable, except for the failure state. To avoid costly failures, the system is monitored by a χ^2 chart. The observation process stochastically related to the machine condition is assumed to be multivariate, normally distributed. When the chart signals, full inspection is performed to determine the actual machine condition. The system can be preventively replaced at a sampling epoch and must be replaced upon failure; preventive replacement costs less than failure replacement. The objective is to find the optimal control chart parameters that minimize the long-run average maintenance cost per unit time. For the economic-statistical design, an additional constraint guaranteeing the occurrence of the true alarm signal on the chart before failure with given probability is considered. For both designs, the objective function is derived using renewal theory.

Keywords: Maintenance; Multivariate statistics; SPC chart design; Cost modeling; Constraints satisfaction

1. Introduction

Maintenance is defined as the combination of all technical and associated administrative actions intended to retain a machine system in a state in which it can perform its required function. Several types of maintenance policies have been considered in the literature, e.g. corrective maintenance, age-based maintenance and condition-based maintenance (CBM). CBM is the maintenance policy in which preventive maintenance is triggered after identifying a symptom of impending failure with the aid of condition-monitoring techniques.

For CBM optimization problems, it is always assumed that the true states of the system are not observable and only partial information is available from regular condition monitoring or sampling. The observation process is stochastically related to the unobservable machine state, so that condition monitoring for maintenance purposes is similar to quality control. Several kinds of CBM models have appeared in the maintenance literature,

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Nomenclature

$q_{i,j}$	the instantaneous transition rate of the state process
$\mathbf{Q}$	the transition rate matrix of the state process
$\lambda_1 = q_{01}$	the transition rate of the system from state 0 to state 1
$\lambda_2 = q_{12}$	the transition rate of the system from state 1 to state 2
$\lambda_3 = q_{02}$	the transition rate of the system from state 0 to state 2
$\upsilon_0 = \lambda_1 + \lambda_3$	1/mean time the machine is in state 0
$\upsilon_1 = \lambda_2$	1/mean time the machine is in state 1
$p_{i,j}$	the transition probability of the embedded Markov process ($p_{ij} = q_{ij}/\upsilon_i, i \neq j$)
$\mathbf{P}$	the transition probability matrix
$p_{i,j}(t)$	the transition probability of the continuous time Markov process
$\mathbf{P(t)}$	the transition probability matrix of the continuous time Markov process
τ_0	sojourn time of the system in state 0 (random variable $\sim\exp(\upsilon_0)$)
τ_1	sojourn time of the system in state 1 (random variable $\sim\exp(\upsilon_1)$)
(μ_0, Σ_0)	the parameters of the multivariate normal distribution of the observation process when the system is in-control
(μ_1, Σ_0)	the parameters of the multivariate normal distribution of the observation process when the system is in the warning state 1
δ	the parameter of the non-central chi-square distribution
k	the dimensionality of the multivariate observation process
T_a	the time to the first alarm on the chart (false alarm or true alarm)
T_f	the time to the system failure
C_a	inspection cost
C_p	preventive replacement cost (C_a is not included)
C_f	failure replacement cost (the necessary inspection cost is included)
C_S	sampling cost
C_0	operating and maintenance cost rate when the system is in state 0
C_1	operating and maintenance cost rate when the system is in state 1
h	sampling interval (decision variable)
UCL	upper control limit of the χ^2 control chart (decision variable)
α	type I error of the chi-square control chart (determined by UCL)
β	type II error of the chi-square control chart (determined by UCL)
$C(\mathrm{UCL}, h)$	objective function, expectation of the long-run average cost per unit time
AC	long-run average cost per unit time
CL	process cycle length
CC	process cycle cost

such as a proportional hazards model in Makis and Jardine (1992), a random coefficient regression model in Lu and Meeker (1993), a counting-process model in Aven (1996), a state-space model in Christer et al. (1997), an optimal-stopping model in Makis et al. (1998), and a hidden Markov model in Makis and Jiang (2003), among others. However, none of the above existing CBM models deals with a multivariate observation process.

Due to the availability of advanced condition-monitoring technologies that are able to collect and store large amount of process data on-line, multivariate observations are available in modern manufacturing. For better CBM optimization, we should consider a multivariate observation process which is both cross and auto-correlated. CBM model development for multivariate observations is a challenging topic. Multivariate modeling of oil data for CBM purposes in our previous research (Wu and Makis, submitted for publication) showed very good results and further motivated our interest in applying multivariate statistical process control (SPC) concepts and methodologies in CBM optimization. In this paper, we consider the problem of designing a multivariate control chart for equipment condition monitoring and maintenance decision-making.