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Optimal scheduling of parallel machines with constrained resources

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Abstract

This paper analyzes a manufacturing system consisting of parallel machines, which produce one product-type with controllable production rates subject to continuously-divisible, time-dependent resources. The objective is to produce the required amount of product-type units by a due date while minimizing inventory, backlog and production related costs over a production horizon. With the aid of the maximum principle, a number of analytical rules of the optimal scheduling is derived whereby the continuous-time scheduling is reduced to discrete sequencing and timing. As a result, a polynomial-time algorithm is developed for solving the problem.

Keywords: Scheduling; Systems dynamics; Constrained resources

1. Introduction

The focus of this paper is a deterministic production system consisting of parallel machines which share resources to produce a number of units of the same product-type in response to demand. Similar to the systems considered by many authors in recent years [1–7], the system considered here includes a buffer with unlimited capacity placed after the machines for the product-type units. If cumulative production of the product exceeds cumulative demand, buffer carrying or inventory costs are incurred. If, on the other hand, cumulative demand exceeds cumulative production, backlog costs are incurred. In addition to inventory related costs, production costs are incurred if the machines are not idle. Given the constrained resources,

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the objective is to find production levels over time such that inventory, backlog as well as production costs are minimized.

One approach, modeling production systems as control problems, was introduced by Kimemia and Gershwin [3] for a system with random machine breakdowns. Two numerical approaches for similar deterministic systems have been developed in [5,7] to cope with continuous-time, multiple-machine production and project scheduling characterized by continuously-divisible constrained resources and arbitrary dynamic demands. Specifically, the former presents an algorithm for dynamic assignment while the latter discusses similarities with the resource constrained production and project scheduling models suggesting an adaptation and numerical method suitable for both production and project scheduling. These general approaches possess two major drawbacks. First, they are based on the gradient projection method to approximate the optimal solution in pseudo-polynomial time, thereby limiting the scale of the problems. Secondly, the accuracy of the solution found is very difficult to estimate and therefore to guarantee. As an alternative to the general approaches, increasing attention is being paid to special cases whereby production systems can be studied and solved analytically rather than numerically. These cases are usually characterized by a single machine producing several product-types given constant demand [1,4,2]. The properties of optimal schedules with continuously-divisible, doubly constrained resources were developed in [8] with the objective of minimizing project duration.

In this paper, unlike the above mentioned analytical works which relied on constant demand and levels of resource usage over time, demand and available resources are piece-wise constant. In other words, the product units are requested at one point in time, a due date, while the level of the available shared resource changes with time in an arbitrary, step-wise manner.

As an example of the real production system that can be modeled in this way, consider a typical fruit juice blending process. At the first stage of the process fruit concentrates are pumped into tanks and stirred. Next, the juice is pumped to continuous parallel blenders to add water, and in some cases sugar, to the concentrate. The output of the blenders is pumped into buffer tanks. There are several shared resources in the system. Many tanks share the same pipes, pumps, cleaning and processing equipment. Moreover, periodic cleaning operations, preventive maintenance and high priority orders make the availability of these resources time dependent. Therefore, production scheduling is of significant importance for such a system. It normally takes place on a weekly basis to meet juice demands as closely as possible by allocating blenders with respect to their utilization/production rates and available shared resource. This example will be further employed in the paper to illustrate the approach.

With the aid of the maximum principle, a number of analytical rules are derived for optimal selection of the machines, their production rates, sequencing and timing. Consequently, the continuous-time scheduling problem is reduced to a combinatorial search for a limited number of switching time points. As a result, the two drawbacks mentioned above are overcome: the solution is obtained in strictly polynomial time and its accuracy is guaranteed. Special cases, when the switching points can be located analytically rather than combinatorially are also discussed and the complexity estimates are derived.

2. Statement of the problem

Consider a production system which produces units of a single product-type in response to demand $D(t)$ for this product-type. The production system consists of N parallel machines and a buffer placed after the machines to hold completed units of the product-type. The maximal production rate of machine n is denoted as U_n. This system represents a cumulative flow of the product-type units through the machines and the buffer:

$$\dot{X}(t) = \sum_n U_n u_n(t), \quad X(0) = 0, \tag{1}$$