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Discrete Optimization

Some inverse optimization problems under the Hamming distance

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Abstract

Given a feasible solution to a particular combinatorial optimization problem defined on a graph and a cost vector defined on the arcs of the graph, the corresponding inverse problem is to disturb the cost vector such that the feasible solution becomes optimal. The aim is to optimize the difference between the initial cost vector and the disturbed one. This difference can be measured in several ways. We consider the Hamming distance measuring in how many components two vectors are different, where weights are associated to the components. General algorithms for the bottleneck or minimax criterion are described and (after modification) applied to the inverse minimum spanning tree problem, the inverse shortest path tree problem and the linear assignment problem.

Keywords: Combinatorial optimization; Inverse optimization; Minimum spanning tree; Shortest path; Linear assignment

1. Introduction

Inverse optimization can be described as the problem to find the best changes in a given cost function (or vector), as to make a given *feasible* solution *optimal*. The criterion of inverse optimization can be the sum of the proposed changes as, e.g., in Ahuja and Orlin (2001). In this paper we consider the weighted Hamming distance and the minimax Hamming distance. He et al. (2001) introduced the *Hamming distance* in the context of inverse optimization problems to evaluate the modifications of the cost function. By this measure, one pays a fixed charge per cost modification (depending on the related solution element, but) irrespective

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of the extent of the modification. In applications where the Hamming distance represents a time, the minimax criterion measures the time to complete a (simultaneous) change of, say, edge-lengths in a network.

Applications of inverse optimization can be characterized as follows (Yang and Zhang, 1999): often optimization models depend on a set of estimated parameters, such as costs, lengths or capacities. Due to frequent use of the model, it is important to know the estimated values more accurately. The optimal solutions of the model under certain circumstances, known by experiments or by experience, can be used to adjust these estimations.

Ahuja and Orlin (2001) have shortly reviewed applications in diverse areas:

- Geophysical sciences, see, e.g., Burton and Toint (1992, 1994) studying inverse shortest path problems in seismic tomography to predict the movement of earthquakes;
- Medical imaging in X-ray tomography where a CT-scan of a body part is exploited to estimate its dimensions given other information about the body;
- Imposing toll in transportation networks, in order to enforce the use of its optimal sub routes according to the policy makers' view of the network.

In the context of high-speed Asynchronous Transfer Mode (ATM) networks Faragó et al. (2003) have the aim to combine the simplicity of fixed routing and some advantages of a dynamic scheme to obtain reliable and stable self-configuring systems. They apply inverse shortest path problems to obtain so-called *Administrative Weights*. These link weights aggregate the potentially complex link state information, thus simplifying the route selection to a shortest path problem; as a result the traffic in the network can be directed similarly to the fixed routing paradigm, without the need to maintain large routing tables.

For some problem types the inverse version of the problem is equivalent to solving a problem of the same kind under a min sum criterion, see further, e.g., Ahuja and Orlin (2001). The complexity status of a problem and the inverse version can differ, as Cai et al. (1999) showed for the polynomial Center Location Problem, with the inverse version that is NP-hard.

Zhang and He (2003) consider the Hamming distance applied to the inverse minimax minimum spanning tree problem. They also describe an algorithm for the inverse sum minimum spanning tree problem under the Hamming distance; it is based on a max flow min cut result and has a large complexity. Section 2 considers the inverse minimax problem under the Hamming distance for combinatorial problems in a network and solution schemes are formulated. Section 3 treats the inverse minimax minimum spanning tree problem and shows that it is solvable in $O(n^2)$ with n the number of nodes, thus improving the known complexity $O(mn)$ with m the number of edges.

In Section 4 we derive an efficient algorithm for the inverse minimum spanning tree problem under the criterion of minimizing the total number of cost modifications. In Section 5 we study the minimax inverse minimum cost flow problem, specializing to the inverse shortest path tree problem and the inverse linear assignment problem.

Finally, we give some remarks and conclusions.

2. The minimax inverse problem

To make the problem description more precise we introduce some notation and some notions. In this section and the following sections we consider a combinatorial optimization problem in a network: a graph $G = (V, E)$ with V the set of n nodes, E the set of m arcs or edges, $E = \{e_1, e_2, \ldots, e_m\}$, and there is given a vector $c \in \Re^m$ of costs on E.