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Continuous Optimization

Mixed symmetric duality in non-differentiable multiobjective mathematical programming ☆

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Abstract

Two mixed symmetric dual models for a class of non-differentiable multiobjective nonlinear programming problems with multiple arguments are introduced in this paper. These two mixed symmetric dual models unify the four existing multiobjective symmetric dual models in the literature. Weak and strong duality theorems are established for these models under some mild assumptions of generalized convexity. Several special cases are also obtained.

Keywords: Symmetric duality; Non-differentiable nonlinear programming; Generalized convexity; Support function

1. Introduction

Dorn [7] introduced symmetric duality in nonlinear programming by defining a program and its dual to be symmetric if the dual of the dual is the original problem. The symmetric duality for scalar programming has been studied extensively in the literature; one can refer to Dantzig et al. [5], Devi [6], Mishra [11,12], Mond [15], Mond and Weir [17].

Mond and Schechter [16] studied non-differentiable symmetric duality for a class of optimization problems in which the objective functions consist of support functions. Following Mond and Schechter [16], Chen [4], Hou and Yang [8], and Yang et al. [21], studied symmetric duality for such problems.

Weir and Mond [20] presented two models for multiobjective symmetric duality. Several authors, such as the ones of [1,2,4,9,10,13,14,18], studied multiobjective second and higher order symmetric duality, motivated by Weir and Mond [20].

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Very recently, Yang et al. [22] presented a mixed symmetric dual formulation for a non-differentiable nonlinear programming problem. Bector et al. [3] introduced a mixed symmetric dual model for a class of nonlinear multiobjective programming problems. However, the models given by Bector et al. [3] as well as by Yang et al. [22] do not allow the further weakening of generalized convexity assumptions on a part of the objective functions.

In this paper, we introduce two models of mixed symmetric duality for a class of non-differentiable multiobjective programming problems with multiple arguments. The first model is a multiobjective version of the model given by Yang et al. [22]. However, the second model is new. Mixed symmetric duality for this model has not been given so far by any other author. The advantage of the second model over the first one is that it allows further weakening of convexity on the functions involved. We establish weak and strong duality theorems for these two models and discuss several special cases of these models. The results of Yang et al. [22] as well as that of Bector et al. [3] are particular cases of the results obtained in the present paper.

2. Preliminaries

For x, $y \in R^n$, by $x \leqq y$ we mean $x_i \leqq y_i$ for all i, $x \leqslant y$ means $x_i \leqq y_i$ for all i and $x_j < y_j$ for at least one j, $1 \leqq j \leqq n$. By $x < y$ we mean $x_i < y_i$ for all i and by $x \nleqslant y$ we mean the negation of $x \leqslant y$.

Let $f(x,y)$ be real valued twice differentiable function defined on $R^n \times R^m$. Let $\nabla_x f(\bar{x},\bar{y})$ and $\nabla_y f(\bar{x},\bar{y})$ denote the partial derivatives of $f(x,y)$ with respect to x and y at $(\bar{x},\bar{y})$. The symbols $\nabla_{xy} f(\bar{x},\bar{y})$, $\nabla_{yx} f(\bar{x},\bar{y})$ and $\nabla_{y^2} f(\bar{x},\bar{y})$ are defined similarly. Consider the following multiobjective programming problem (VP):

$$\min(f_1(x), f_2(x), \ldots, f_p(x))$$
$$\text{s.t.} \quad h(x) \leqq 0,$$

where $f_i: R^n \to R, i = 1, 2, \ldots, p$ and $h: R^n \to R^m$.

Let us denote the feasible region of problem (VP) by $X_0 = \{x \in R^n : h_j(x) = 0, j = 1, 2, \ldots, m\}$. For problem (VP), an efficient solution and a properly efficient solution are defined as follows.

Definition 1. A feasible solution x^0 is said to be an efficient solution for (VP) if there exists no other feasible solution x such that

$$f(x) \leqslant f(x^0).$$

Let C be a compact convex set in R^n. The support function of C is defined by

$$s(x \mid C) = \max\{x^{\mathrm{T}} y : y \in C\}.$$

A support function, being convex and everywhere finite, has a subdifferential [19], that is, there exists $z \in R^n$ such that

$$s(y \mid C) \geqslant s(x \mid C) + z^{\mathrm{T}}(y - x), \quad \forall y \in C.$$

The subdifferential of $s(x \mid C)$ is given by

$$\partial s(x \mid C) = \{z \in C : z^{\mathrm{T}} x = s(x \mid C)\}.$$

For any set $D \subset R^n$, the normal cone to D at a point $x \in D$ is defined by

$$N_D(x) = \{y \in R^n : y^{\mathrm{T}}(z - x) \leqslant 0, \ \forall z \in D\}.$$

It is obvious that for a compact convex set C, $y \in N_C(x)$ if and only if $s(y \mid C) = x^{\mathrm{T}} y$, or equivalently, $x \in \partial s(y \mid C)$.

The following definitions will be needed in the sequel.

Definition 2. Let $X \subset R^n$. A functional $F: X \times X \times R^n \to R$ is said to be sublinear with respect to its third argument if for any x, $y \in X$

(A) $F(x,y;a_1 + a_2) \leqslant F(x,y;a_1) + F(x,y;a_2)$ for any $a_1, a_2 \in R^n$;
(B) $F(x,y;\alpha a) \leqslant \alpha F(x,y;a)$ for any $\alpha \in R_+$ and $a \in R^n$.