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Discrete Optimization

A branch and bound algorithm for the one-machine scheduling problem with minimum and maximum time lags

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Abstract

We consider the one-machine scheduling problem with minimum and maximum time lags while minimizing the makespan. This problem typically arises in a manufacturing environment where the next job has to be carried out within a specific time range after the completion of the immediately preceding job. We describe a branch and bound algorithm, based on the input and output of a clique and the relevant propositions, for finding the optimal waiting times. The computational experiments give promising results, showing whether a given instance is feasible or infeasible. With the proposed branch and bound algorithm we can either find an optimal schedule or establish the infeasibility within an acceptable run time.

Keywords: Scheduling; One machine; Makespan; Branch and bound algorithm; Minimum and maximum time lags

1. Introduction

In this paper, we discuss a one-machine problem with minimum and maximum time lags between a pair of jobs. The objective is to minimize the makespan. A minimum time lag specifies that a job can only start after the preceding job has already been processed for some period of time. The maximum time lag specifies the time after the preceding job has been started when a new job should be started. The studied problem arises in the context of the manufacturing environment, where a job has to be carried out within a limited time range after the completion of the immediately preceding job or for job shop scheduling with minimum and maximum time lag constraints. Fig. 1 shows an example of the studied problem for machine 1, which originates from a 3×3 job shop problem. Node (j, i) denotes the processing of job j on machine i. The range attached to the arc is the waiting time. The two numbers indicate the minimum and maximum time lags less the processing time of the preceding job. The machine 1 nodes are circled by the dashed line. Assume that there are n jobs that need to be processed on machine 1 and their job sequence is not yet determined. To sequence n jobs is the same as determining the direction of each arc so that the waiting time falls within the range attached to the arc; in other

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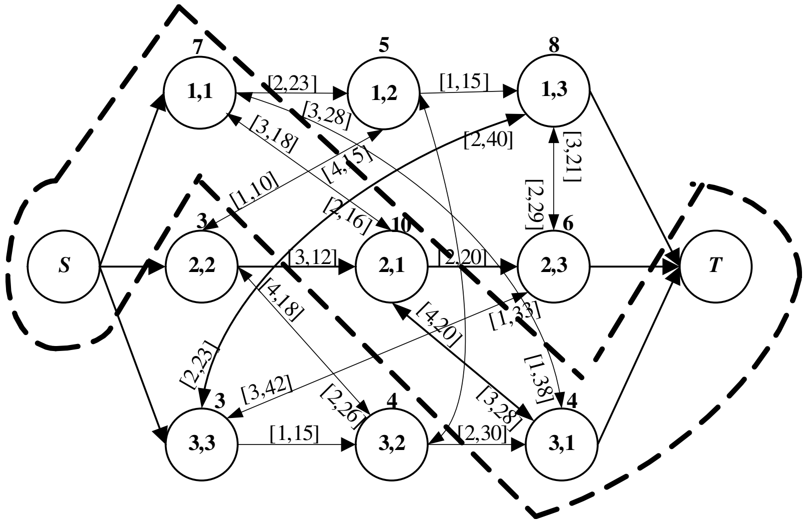

Fig. 1. A 3×3 job shop. Operation (j, i) indicates the processing of job j on machine i.

words, the starting time of each job should satisfy the minimum and maximum time lags specified while still minimizing the makespan.

Temporal constraints such as minimum and maximum time lags occur in practical applications such as food production, chemical production and steel production (Yang and Chern, 1995). For example the chance of food contamination during the production process, and the absorption of air borne particulates during chemical production process can be reduced or eliminated by limiting the waiting time. Fabrication operations are easier to carry out before the temperature of heated steel drops too much. The hoist scheduling problem is another area with research issues related to temporal constraints. This type of problem originated by limits during the printed circuit board (PCB) process due to the chemical reaction time (Lei and Wang, 1991; Yih et al., 1993; Armstrong et al., 1994; Shapiro and Nuttle, 1998). The reaction time in the PCB chemical bath should be kept within a certain time range; otherwise a defective product will be produced. In the PCB case, the temporal constraint is on the operation. Su (2003) reported that, in a wafer fabrication process, the waiting time in a furnace tube after an operation must be limited in order to prevent the absorption of particulates in air.

According to Baker (1974), the one-machine problem is a building block in the development of a comprehensive understanding of complicated systems such as job shops. In Carlier's (Carlier, 1982) study of the one-machine problem, he developed a branch and bound algorithm. This algorithm was based on a critical job and a critical set of jobs found by the Schrage algorithm for each branching node, and optimally solved the problem. These findings of Carlier (1982) were used by Adams et al. (1988), Brucker et al. (1994a,b), Carlier and Pinson (1989, 1990, 1994), and Dauzere-Peres and Lasserre (1993) as the base for solving job shop problems. It can thus be seen that the study of the one-machine problem is important for complicated systems, since its results provide a bound to the makespan, and are part of the fundamental structure for analyzing the job shop problem.

Scheduling problems with minimum and maximum time lags are particularly important in the area of resource-constraint project scheduling (Neumann and Zhan, 1995; Brinkmann and Neumann, 1996; Neumann and Schwindt, 1997; Franck et al., 2001; Heilmann, 2003; Neumann et al., 2003). For example, Neumann and Zhan (1995), and Brinkmann and Neumann (1996) proposed heuristics based on priority-rule methods for this type of problem. Neumann and Schwindt (1997) showed that, in addition to minimum ones, maximum time lags can also be modeled by a cyclic activity-on-node network. They discussed the application of the problem in make-to-order production. Franck et al. (2001) considered the problem of project scheduling with calendar constraints. They proposed using a polynomial time algorithm for finding calendar-feasible