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O.R. Applications

Cycles of violence: A dynamic control analysis ☆

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Abstract

We introduce and analyze a simple model of cycles of violence in which oscillations are generated when surges in lethal violence shrink the pool of active violent offenders. Models with such endogenously induced variation may help explain why historically observed trends in violence are generally not well correlated with exogenous forcing functions, such as changes in the state of the economy. The analysis includes finding the optimal dynamic trajectory of incarceration and violence prevention interventions. Those trajectories yield some surprising results, including situations in which myopic decision makers will invest more in prevention than will far-sighted decision makers.

Keywords: Cost benefit analysis; Cycles of violence; Hopf bifurcation; Optimal control; OR in government

1. Introduction

Rates of crime and violence can vary substantially over time, but the variations are not always well-correlated with indicators of "root causes". This paper introduces a non-linear model of an endogenous feedback affecting violent offenders that offers a novel possible explanation for such a lack of correlation.

There is less than perfect correlation between crime rates and their "root causes". As James Q. Wilson (1975) observed many years ago, crime soared in the US in the 1960s even though it was a decade of prosperity that witnessed aggressive social interventions such as the "war on poverty". Wilson dubbed this crime amidst plenty the "paradox of the 1960s", but the paradox is not limited to the 1960s. During the Reagan-era economic expansion the unemployment rate fell steadily from

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a peak of 9.7% to 5.3%, but there was essentially no net change in the number of homicides.[1] In other periods, such as the 1990s, crime and economic indicators move in tandem. Overall, though, between Wilson's writing in 1975 and 2001 there was actually a slight negative correlation between the rates of unemployment and violence in the US.[2]

Lack of correlation does not stem from lack of variation. Homicide rates in the US doubled between 1960 and 1980 (from 5.1 to 10.2 per 100,000 population) then fell back almost to its original levels by 2000 (to 5.5 per 100,000). Nor do such simple statistics imply that root causes are irrelevant. Rather, the amplitude of variation and lack of correlation with traditional macro explanatory variables suggests the possibility of interesting internal dynamics and possibly indirect or lagged relationships between root causes and violence.

A variety of hypotheses along these lines have been advanced, including that violence toward children prompts those children to become violent offenders (Widom and Maxfield, 2001) and that variation in the availability of abortion affects the proportion of children who grow up to be violent offenders (Donohue and Levitt, 2001). Another hypothesis is that violence is inherently self-limiting because crime is most often committed within rather than across social groups. In particular, lethal violence may be perpetrated most often against people who are themselves criminally involved.[3] In this vision of the world, epidemics of violence may burn themselves out by self-induced attrition of violent offenders. This paper proposes a formal model that captures this dynamic and explores some policy implications of trying to control optimally the cycles of violence that can occur in this model.

In a continuous time dynamic model, cycles cannot emerge with a single state variable, so the number of violent offenders must be augmented by some other quantity for non-monotonic variation to emerge. One possibility is that lethal interactions between violent offenders are much more likely within a general climate or expectation of violence than when the expectation is one of amicable co-existence and that this climate of violence is driven by recollection of actual instances of violence, not merely by counts of the current number of potential offenders. In other words, it is possible for a large number of potentially violent offenders to co-exist without killing each other in great numbers if there is a tradition of non-lethal interactions, but the expectation of violent interactions could itself be a violence promoter.

An expectation of violence can be a self-fulfilling prophecy for any of several reasons. For example, if a violence-prone individual perceives the world to be very risky, the individual might be more likely to carry a weapon, which in turn increases the likelihood confrontations will escalate to lethal violence (cf. Blumstein et al., 2000). Likewise, if one expects an adversary to attack, that increases the incentive to commit a preemptive first strike. Also, high levels of background violence might affect behavioral norms. In a society in which violence is rare, an individual might respond to an insult or challenge with words or fists, believing that more lethal actions are uncalled for.

This suggests analyzing a model with two states: the number of potentially violent offenders (x) and the perceived climate or expectation of violence (y). (Note that in what follows, whenever referring to the state variable y, we will simply call it the "climate of violence".) One of the flows out of the population of potentially violent individuals is proportional (with proportionality constant α) to the square of the number of such individuals (as in predator–prey interactions, except that the predator and prey are the same species in this case) and to the climate of violence. Every such exit (representing a death) also generates an inflow to the climate of violence. Violent interactions between the offenders (x) and the general public (whose population is assumed to be constant) are also assumed to be both promoted by and contribute to this climate of violence (y), so there is a second inflow to y that is proportional (with proportionality constant β) to xy.

To focus attention on this dynamic, the rest of the model will be kept quite simple, with just a simple growth model for the offenders and exponential decay of the climate of violence (at a rate δ). In particular, individuals join the pool of violent offenders at a constant rate of k individuals per unit time, perhaps because a fixed proportion of every birth

[1] There was a slight increase in the *number* of homicides from 21,010 in 1982 to 21,500 in 1989, but a slight decrease in the homicide *rate* per capita.

[2] Correlation −0.069. Unemployment rates downloaded from the Bureau of Labor Statistics web site; annual rates are simple average of the year's four seasonally adjusted quarterly rates. Crime rates downloaded from the Bureau of Justice Statistics web site (http://bjsdata.ojp.usdoj.gov/dataonline/Search/Crime).

[3] This possibility was suggested to the authors by Alfred Blumstein, personal communications.