Available online at www.sciencedirect.com
SCIENCE DIRECT®
European Journal of Operational Research 169 (2006) 189–201
EUROPEAN JOURNAL OF OPERATIONAL RESEARCH
www.elsevier.com/locate/ejor

Stochastics and Statistics

Random fuzzy renewal process

Ruiqing Zhao *, Wansheng Tang, Huaili Yun

Institute of Systems Engineering, Tianjin University, Tianjin 300072, China

Received 3 November 2003; accepted 19 April 2004
Available online 3 August 2004

Abstract

This paper attempts to discuss a random fuzzy renewal process based on random fuzzy theory. The interarrival times are characterized as nonnegative random fuzzy variables which is a more reasonable consideration in the real world. Under this setting, the rate of the random fuzzy renewal process is discussed and a random fuzzy elementary renewal theorem is presented. Furthermore, the expected value of renewals in an arbitrary interval is investigated and Blackwell's theorem in random fuzzy sense is also established.

Keywords: Fuzzy sets; Stochastic process; Renewal process; Random fuzzy variable; Interarrival time

1. Introduction

Based on probability theory, a lot of research has been reported in the area of stochastic renewal process such as Allan [1], Alsmeyer [2], Ferreira [3], Ross [14], Takis [16] and Visaggio [17]. In stochastic renewal process, an underlying assumption is that the interarrival times are deemed to be random variables. However, consideration of random uncertainty alone cannot satisfactorily evaluate a process. From a practical viewpoint, the fuzziness and randomness in one process are often mixed up with each other and it is not easy to distinguish them. In such a case, fuzziness and randomness are required to be considered simultaneously.

There are two tools to deal with this kind of phenomena. One is the fuzzy random theory first introduced by Kwakernaak [5,6]. Roughly speaking, a fuzzy random variable is mathematical description for a fuzzy random phenomenon and defined as a measurable function from a probability space to a collection of fuzzy sets. Based on the fuzzy random theory, some fuzzy random processes have been considered by several authors. Hwang [4] investigated a renewal process in which the interarrival times were considered as

* Corresponding author.
E-mail addresses: rzhao@orsc.edu.cn (R. Zhao), wstang@eyou.com (W. Tang), yunhuaili1971@eyou.com (H. Yun).

0377-2217/$ - see front matter © 2004 Elsevier B.V. All rights reserved.
doi:10.1016/j.ejor.2004.04.049

independent and identically distributed fuzzy random variables and a theorem for the fuzzy rate of fuzzy random renewal process was provided. Popova and Wu [13] considered a renewal reward process with fuzzy random interarrival times and rewards. Also, the long-run average fuzzy reward per unit time was stated in [13]. The other is the random fuzzy theory presented by Liu [7]. In brief a random fuzzy variable is a function from a possibility space to a collection of random variables. An expected value operator of random fuzzy variable was introduced by Liu and Liu [11]. Both fuzzy random theory and random fuzzy theory offer powerful tools for describing and analyzing the uncertainty of combining randomness and fuzziness.

In this paper, we shall develop a theory helpful in the study of renewal process with random fuzzy interarrival times. In Sections 2 and 3, we recall some basic concepts and results about fuzzy variables and random fuzzy variables, respectively. In Section 4, a renewal process in which the interarrival times are characterized as random fuzzy variables is discussed and the random fuzzy elementary renewal theorem as well as Blackwell's theorem in random fuzzy sense is established.

2. Fuzzy variables

We start this section by giving some concepts and properties of fuzzy variables, which will be used in the rest of this paper. Let ξ be a fuzzy variable on a possibility space $(\Theta, P(\Theta), \mathrm{Pos})$ (for the concept of the possibility space, see [7] and [12]), where Θ is a universe, $P(\Theta)$ is the power set of Θ and Pos is a possibility measure defined on $P(\Theta)$.

Based on the *possibility measure* Pos, the *necessity* (Nec) and *credibility* (Cr) of a fuzzy event $\{\xi \geqslant r\}$ can be expressed by

$$\begin{aligned} &\mathrm{Nec}\{\xi \geqslant r\} = 1 - \mathrm{Pos}\{\xi < r\}, \\ &\mathrm{Cr}\{\xi \geqslant r\} = \frac{1}{2}(\mathrm{Pos}\{\xi \geqslant r\} + \mathrm{Nec}\{\xi \geqslant r\}), \end{aligned} \tag{1}$$

respectively.

Definition 1 (Liu [9]). The fuzzy variables $\xi_1, \xi_2, \ldots, \xi_n$ are said to be independent if and only if

$$\mathrm{Pos}\{\xi_i \in B_i,\ i = 1, 2, \ldots, n\} = \min_{1 \leqslant i \leqslant n} \mathrm{Pos}\{\xi_i \in B_i\} \tag{2}$$

for any sets $B_1, B_2, \ldots, B_n$ of $\Re$.

Definition 2 (Liu [9]). The fuzzy variables $\xi_1, \xi_2, \ldots, \xi_n$ are said to be identically distributed if and only if

$$\mathrm{Pos}\{\xi_i \in B\} = \mathrm{Pos}\{\xi_j \in B\},\ i, j = 1, 2, \ldots, n, \tag{3}$$

for any set B of $\Re$.

Definition 3 (Liu and Liu [10]). Let ξ be a fuzzy variable. The expected value $E[\xi]$ is defined as

$$E[\xi] = \int_0^\infty \mathrm{Cr}\{\xi \geqslant r\}\mathrm{d}r - \int_{-\infty}^0 \mathrm{Cr}\{\xi \leqslant r\}\mathrm{d}r \tag{4}$$

provided that at least one of the two integrals is finite. Especially, if ξ is a nonnegative fuzzy variable, then $E[\xi] = \int_0^\infty \mathrm{Cr}\{\xi \geqslant r\}\mathrm{d}r$.

Definition 4 (Liu [7]). Let ξ be a fuzzy variable and $\alpha \in (0,1]$. Then

$$\xi'_\alpha = \inf\{r \mid \mathrm{Pos}\{\xi \leqslant r\} \geqslant \alpha\} \quad \text{and} \quad \xi''_\alpha = \sup\{r \mid \mathrm{Pos}\{\xi \geqslant r\} \geqslant \alpha\} \tag{5}$$

are called the α-pessimistic and α-optimistic values of ξ, respectively.