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Discrete Optimization

A method for finding well-dispersed subsets of non-dominated vectors for multiple objective mixed integer linear programs

John Sylva [a], Alejandro Crema [b,*]

[a] Departamento de Matemáticas Aplicadas, Facultad de Ingeniería, Universidad Central de Venezuela, Venezuela
[b] Facultad de Ciencias, Escuela de Computación, Universidad Central de Venezuela, Apdo. 47002, Caracas 1041-A, Venezuela

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Abstract

We present an algorithm for generating a subset of non-dominated vectors of multiple objective mixed integer linear programming. Starting from an initial non-dominated vector, the procedure finds at each iteration a new one that maximizes the infinity-norm distance from the set dominated by the previously found solutions. When all variables are integer, it can generate the whole set of non-dominated vectors.

Keywords: Integer programming; Multiple objective programming; Parametric programming

1. Introduction

Generating non-dominated vectors and efficient solutions in multiple objective mathematical programming has been a field of interest both as a theoretical problem and as a part of multiple criteria decision making procedures.

Several procedures have been developed for generating the non-dominated set for multiple objective linear programming (MOLP) [4,5,7], multiple objective integer linear programming (MOILP) [3,11,16,17], and multiple objective mixed integer linear programming (MOMILP) [10] problems. However, these methods have a considerable computational cost and, due to the great number of solutions, their results cannot be used without relying on filtering procedures. There are also interactive procedures for these problems [1,8,14] that try to overcome the computational cost but they may need filtering too in order to get well-dispersed results.

The method we present in this paper was originally conceived as a variant of the procedure we proposed in [15]. That method, based on [9] could only generate the whole set of non-dominated vectors for MOILP

* Corresponding author. Fax: +58 2 60 52 168.
E-mail addresses: jsylva@reacciun.ve (J. Sylva), acrema@kuaimare.ciens.ucv.ve (A. Crema).

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problems with integer entries in the cost matrix. Our original goal was to overcome this limitation, but we found however a more useful feature of the new procedure, namely, that it generates at each iteration the vector that maximizes the infinity-norm distance to the set dominated by all the previously found vectors. This characteristic allow us to obtain uniform representations (as defined in [12]) of the set of non-dominated vectors using partial runs of the procedure without relying on filtering techniques. On the contrary, in our first procedure the solutions are generated in decreasing order of some fixed weighted objective function. This difference is important if we consider that a full enumeration of the set of non-dominated vectors is impossible in practice with these methods as the complexity of the integer linear problems grows with the number of solutions generated. On the contrary, partial runs of the procedures are more viable and therefore, the order of generation of the solutions is a critical aspect to take into account when comparing both approaches.

The first method allows us to get better solutions first if we have a good estimate of the decision maker's preferences but when there is little information about these, a representative subset of non-dominated can only be achieved by introducing problem-dependant step parameters and there is no guarantee of uniformity. On the contrary, our new proposition can generate uniform representations by specifying more natural parameters such as the number of elements in the subset or the minimal distance between its elements.

The procedure can be applied to problems with continuous variables such as MOLP and MOMILP problems allowing us to produce well-dispersed samples of non-dominated vectors to these classes of problems.

2. Theoretical basis

The MOMILP problem can be stated as

$$(P): \quad \text{“}\max\text{''}\{Cx : Ax \leqslant b, x \geqslant 0, x_j \in \mathbb{Z} \text{ for } j \in J\},$$

where $C \in \mathbb{R}^{p\times n}$, $A \in \mathbb{R}^{m\times n}$, $b \in \mathbb{R}^m$ and $J \subseteq \{1,\ldots,n\}$. Cx represents p objective functions, $Ax \leqslant b$ represents m linear constraints and x represents n decision variables. J is an index set of the integer variables; two important cases considered are the MOLP problem, where $J = \emptyset$ and the MOILP problem, where $J = \{1,\ldots,n\}$.

The feasible set of problem (P) will be denoted by $F(P)$ and is assumed to be a non-empty bounded set.

Because of conflicting objectives, there is not usually a maximum solution but maximal or non-dominated solutions.

Definition 2.1. A feasible solution x^* to problem (P) is an *efficient solution* iff there is not another feasible x such that $Cx \geqslant Cx^*$ with at least one strict inequality. The resulting criterion vector Cx^* is said to be non-dominated.

A well-known result connecting multiple objective programming and parametric programming is the following [14]:

Theorem 2.2. *If x^* is an optimal solution to the (single objective) problem*

$$\max\{\lambda^t Cx : x \in S\}$$

for some $\lambda \in \mathbb{R}^p$, $\lambda > 0$, then x^* is an efficient solution to problem

$$\text{“}\max\text{''}\{Cx : x \in S\}.$$

Efficient solutions that are optimal to the parametric problem in Theorem 2.2 are said to be *supported efficient solutions*. Unlike MOLP, the reciprocal of this theorem does not hold for MOMILP or MOILP [2] as some efficient solutions (known as *unsupported efficient solutions*) may not be optimal for any $\lambda > 0$.

The following results allow us to find a solution (supported or unsupported) such that its corresponding objective vector is at a maximal distance, accordingly to the infinity-norm, from the region dominated by a finite set of known non-dominated vectors.

In the next two lemmas, we will prove that, given a set $\{x^1,\ldots,x^l\}$ of efficient solutions and an $\hat{x}$, candidate to be efficient, it is possible to calculate the infinity-norm distance from vector $C\hat{x}$ to the set $\{z|z \leqslant Cx^s$, for some $s = 1,\ldots,l\}$ by solving a mixed integer linear programming (MILP) problem.