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Decision Support

Self-consistency of decision rules for group decision making

Takehiro Inohara *

Department of Value and Decision Science, Graduate School of Social and Decision Science and Technology, Tokyo Institute of Technology, 2-12-1 O-okayama Meguro, Tokyo 152-8552, Japan

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Abstract

The author treats, in this paper, a group of decision makers, where each of them already has preference on a given set of alternatives but the group as a whole does not have a decision rule to make their group decision, yet. Then, the author examines which decision rules are appropriate. As a criterion of "appropriateness" the author proposes the concepts of self-consistency and universal self-consistency of decision rules. Examining the existence of universally self-consistent decision rules in two cases: (1) decision situations with three decision makers and two alternatives, and (2) those with three decision makers and three alternatives, the author has found that all decision rules are universally self-consistent in the case (1), whereas all universally self-consistent decision rules have one and just one vetoer in the essential cases in (2). The result in the case (2) implies incompatibility of universal self-consistency with symmetry. An example of applications of the concept of self-consistency to a bankruptcy problem is also provided in this paper, where compatibility of self-consistency with symmetry in a particular decision situation is shown.

Keywords: Game theory; Group decision making; Decision rule; Simple game; Arrow's impossibility theorem

1. Introduction

In this paper, I treat a group of decision makers, where each of them already has preference on a given set of alternatives but the group as a whole does not have a decision rule to make their group decision, yet. Such groups are so realistic, because a decision making problem usually occurs before forming a corresponding decision making group with a decision rule as a whole, rather than after that. For example, newly discovered resources often involve some decision makers, who have not had any interaction with each other before the discovery of the resources, in the same problem of the allocation of the resources. The members in such a group have to select one decision rule from the set of all possible decision rules, in order to select one alternatives from the set of all possible alternatives.

There are a lot of logically possible decision rules for group decision making, and the decision of the group as a whole depends on the decision rule that is adopted. Thus, since each decision maker prefers a decision rule

* Tel./fax: +81 3 5734 3366.
E-mail addresses: inohara@valdes.titech.ac.jp, inotake@yc4.so-net.ne.jp

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that is likely to achieve alternatives that are preferable for him/herself, he/she has preference on the set of all decision rules. That is, an original decision situation, in which the decision makers have to select one from the set of all alternatives, generates a "meta" decision situation, in which they have to select one from the set of all decision rules. An appropriate decision rule should be selected in the "meta" decision situation, so that an appropriate alternative is selected in the original decision situation.

As a criterion of "appropriateness" of a decision rule, I propose the concept of self-consistency and universal self-consistency of decision rules. Self-consistency requires that the decision rule of an original decision situation should select itself when it is applied to the "meta" decision situations of the original decision situations. Moreover, a decision rule is said to be universally self-consistent, if the decision rule is self-consistent independently of the preference of decision makers on alternatives in original decision situations.

Then, I examine the existence of self-consistent and universally self-consistent decision rules. I examine two cases: (1) decision situations with three decision makers and two alternatives, and (2) those with three decision makers and three alternatives. Then, it is found that all decision rules are universally self-consistent in the case (1), whereas all universally self-consistent decision rules have one and just one vetoer in the essential cases in (2). Since decision rules with one and just one vetoer is not desirable for us, the result in the case (2) implies incompatibility of universal self-consistency with symmetry. Moreover, these results suggest that the concepts of self-consistency and universal self-consistency may have some relations with the Arrow's impossibility theorem (see [1; 6, p. 22]), the Gibbard-Satterthwaite theorem (see [3,8; 6, p. 33]), and the theorem on the relationship between the existence of non-empty core and the Nakamura number (see [5; 6, p. 36]).

The structure of this paper is as follows: in the next section, a mathematical framework for treating group decision making situations is provided, based on that for analysis of voting in committees [6]. In Section 3, using the concept of core [6], I introduce a method to generate meta decision situations from original decision situations. Then, in Section 4, definitions of self-consistency and universal self-consistency are provided. Results of analysis are shown in Section 5, and an example of applications of the concept of self-consistency to a bankruptcy problem [2] is given in Section 6. The last section, Section 7, is devoted to conclusions of this paper.

2. Framework

In this section, a mathematical framework for treating group decision making situations is provided, based on that for analysis of voting in committees [6]. Firstly, a decision making group, or simply, a group, is defined.

Definition 1 (*Groups*). A group C is a pair (N, A), where N is the finite set of all decision makers (DMs), and A is the finite set of all alternatives.

A group with a list of preferences of DMs constitutes a meeting.

Definition 2 (*Meetings*). A meeting C_R is a 3-tuple (N, A, R), where $C = (N, A)$ is a group, and R is a list $(R_i)_{i \in N}$ of preferences R_i of each DM $i \in N$ on A.

Let us assume that for $i \in N$, the preference R_i of DM $i \in N$ is a linear ordering on A, that is, R_i satisfies the following four conditions: (i) reflexivity: for $x \in A$, xR_ix, (ii) transitivity: for $x, y, z \in A$, if xR_iy and yR_iz, then xR_iz, (iii) anti-symmetry: for $x, y \in A$, if xR_iy and yR_ix, then $x = y$, and (iv) completeness: for $x, y \in A$, xR_iy or yR_ix. The set of all linear orderings on A is denoted by $L(A)$.

Moreover, for $x, y \in A$, xR_iy means that DM i prefers alternative x to alternative y, or is indifferent between alternative x and alternative y. xI_iy means that xR_iy and yR_ix, that is, DM i is indifferent between state x and state y. By the assumption of anti-symmetry, xI_iy implies $x = y$. xP_iy means that xR_iy and "not (yR_ix)," that is, DM i strictly prefers state x to state y.

Example 1 (*Meetings*). Let N and A be $\{1, 2, 3\}$ and $\{a, b, c\}$, respectively. Let, moreover, $R = (R_i)_{i \in N}$ be $R_1 = [a, b, c]$, $R_2 = [c, a, b]$, and $R_3 = [c, b, a]$, where for $i \in N$ and $x, y, z \in A$, $R_i = [x, y, z]$ means that xP_iy and yP_iz (accordingly, xP_iz). Then, (N, A, R) constitutes a meeting. If $R'_1 = [a, b, c]$, $R'_2 = [b, c, a]$, and $R'_3 = [c, a, b]$, then (N, A, R') becomes another meeting, where $R' = (R'_i)_{i \in N}$.