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Discrete Optimization

Approximation of min–max and min–max regret versions of some combinatorial optimization problems

Hassene Aissi, Cristina Bazgan, Daniel Vanderpooten *

LAMSADE, Université Paris-Dauphine, Place du Marechal de Lattre de Ta., 75775 Paris Cedex 16, France

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Abstract

This paper investigates, for the first time in the literature, the approximation of min–max (regret) versions of classical problems like shortest path, minimum spanning tree, and knapsack. For a constant number of scenarios, we establish fully polynomial-time approximation schemes for the min–max versions of these problems, using relationships between multi-objective and min–max optimization. Using dynamic programming and classical trimming techniques, we construct a fully polynomial-time approximation scheme for min–max regret shortest path. We also establish a fully polynomial-time approximation scheme for min–max regret spanning tree and prove that min–max regret knapsack is not at all approximable. For a non-constant number of scenarios, in which case min–max and min–max regret versions of polynomial-time solvable problems usually become strongly NP-hard, non-approximability results are provided for min–max (regret) versions of shortest path and spanning tree.

Keywords: Min–max; Min–max regret; Approximation; fptas; Shortest path; Minimum spanning tree; Knapsack

1. Introduction

The definition of an instance of a combinatorial optimization problem requires to specify parameters, in particular coefficients of the objective function, which may be uncertain or imprecise. Uncertainty/imprecision can be structured through the concept of *scenario* which corresponds to an assignment of plausible values to model parameters. There exist two natural ways of describing the set of all possible scenarios. In the *interval data case*, each numerical parameter can take any value between a lower and an upper bound. In the *discrete scenario case*, the scenario set is described explicitly. In this case, that we address in this paper, we distinguish situations where the number of scenarios is constant from those where the number of scenarios is non-constant. Kouvelis and Yu [8] proposed the min–max and min–max regret criteria, stemming from decision theory, to construct solutions hedging against parameters variations. The min–max criterion aims at constructing solutions having a good performance in the worst case. The min–max regret criterion, less conservative, aims at obtaining a solution minimizing the maximum deviation, over all

* Corresponding author.
E-mail addresses: aissi@lamsade.dauphine.fr (H. Aissi), bazgan@lamsade.dauphine.fr (C. Bazgan), vdp@lamsade.dauphine.fr (D. Vanderpooten).

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possible scenarios, of the value of the solution from the optimal value of the corresponding scenario.

Complexity of the min–max and min–max regret versions has been studied extensively during the last decade. Kouvelis and Yu [8] established the complexity of min–max and min–max regret versions, for the discrete scenario case, of several combinatorial optimization problems, including shortest path, minimum spanning tree, assignment, and knapsack problems. In general, these versions are shown to be harder than the classical versions. Min–max (regret) versions of polynomial problems usually become weakly NP-hard for a constant number of scenarios, and strongly NP-hard for a non-constant number of scenarios.

In this paper we consider, for the first time in the literature, the approximation of min–max (regret) versions of classical problems like shortest path, minimum spanning tree, and knapsack. For a constant number of scenarios, we establish fully polynomial-time approximation schemes (fptas) for the min–max versions of these problems, using relationships between multi-objective and min–max optimization. The interest of studying these relationships is that fptas, which determine an approximation of the non-dominated set (or Pareto set), have been proposed for the multi-objective version (see, e.g., [3,11,12]). This allows us to derive the existence of fptas for min–max versions of our reference problems. Concerning min–max regret versions, relationships with multi-objective versions still apply but cannot be used to derive the existence of fptas. Using dynamic programming and classical trimming techniques, we construct an fptas for min–max regret shortest path. We also give an fptas for min–max regret spanning tree and prove that min–max regret knapsack is not at all approximable. For a non-constant number of scenarios, non-approximability results are provided for min–max (regret) versions of shortest path and spanning tree. All the results are summarized in Table 1.

After presenting preliminary concepts in Section 2, we investigate the existence of approximation algorithms for our reference problems when the number of scenarios is constant (Section 3), and when it is non-constant (Section 4).

2. Preliminaries

We consider in this paper the class $\mathscr{C}$ of 0–1 problems with a linear objective function defined as

$$\begin{cases} \min \sum_{i=1}^{n} c_i x_i, & c_i \in \mathbb{N}, \\ x \in X \subset \{0,1\}^n. \end{cases}$$

This class encompasses a large variety of classical combinatorial problems, some of which are polynomial-time solvable (shortest path problem, minimum spanning tree,...) and others are NP-hard (knapsack, set covering,...). The size of a solution $x \in X$ is the number of variables x_i which are set to 1.

2.1. Min–max, min–max regret versions

Given a problem $\mathscr{P} \in \mathscr{C}$, the min–max (regret) version associated to $\mathscr{P}$ has as input a finite set of scenarios S where each scenario $s \in S$ is represented by a vector $(c_1^s, \ldots, c_n^s)$. We denote by $\mathrm{val}(x,s) = \sum_{i=1}^{n} c_i^s x_i$ the value of solution $x \in X$ under scenario $s \in S$ and by val_s^* the optimal value in scenario s.

The min–max optimization problem corresponding to $\mathscr{P}$, denoted by Min–Max $\mathscr{P}$, consists of finding a solution x having the best worst case value across all scenarios, which can be stated as

$$\min_{x \in X} \max_{s \in S} \mathrm{val}(x,s).$$

Given a solution $x \in X$, its *regret*, $R(x,s)$, under scenario $s \in S$ is defined as $R(x,s) = \mathrm{val}(x,s) - \mathrm{val}_s^*$. The *maximum regret* $R_{\max}(x)$ of solution x is then defined as $R_{\max}(x) = \max_{s \in S} R(x,s)$.

The min–max regret optimization problem corresponding to $\mathscr{P}$, denoted by Min–Max Regret $\mathscr{P}$, consists of finding a solution x minimizing the maximum regret $R_{\max}(x)$ which can be stated as

Table 1
Approximation results for min–max and min–max regret versions

	Constant		Non-constant	
	Min–max	Min–max regret	Min–max	Min–max regret
Shortest path	fptas	fptas	Not $(2-\varepsilon)$ approx.	Not $(2-\varepsilon)$ approx.
Min spanning tree	fptas	fptas	Not $\left(\frac{3}{2}-\varepsilon\right)$ approx.	Not $\left(\frac{3}{2}-\varepsilon\right)$ approx.
Knapsack	fptas	Not at all approx.	Not at all approx.	Not at all approx.