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Multi-objective stochastic programming for portfolio selection

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Abstract

Generally, in the portfolio selection problem the Decision Maker (DM) considers simultaneously conflicting objectives such as rate of return, liquidity and risk. Multi-objective programming techniques such as goal programming (GP) and compromise programming (CP) are used to choose the portfolio best satisfying the DM's aspirations and preferences. In this article, we assume that the parameters associated with the objectives are random and normally distributed. We propose a chance constrained compromise programming model (CCCP) as a deterministic transformation to multi-objective stochastic programming portfolio model. CCCP is based on CP and chance constrained programming (CCP) models. The proposed program is illustrated by means of a portfolio selection problem from the Tunisian stock exchange market.

Keywords: Goal programming; Compromise programming; Chance constrained programming; Chance constrained compromise programming; Portfolio selection

1. Introduction

The basic theory of portfolio selection was initiated by Markowitz (1952). The portfolio selection problem is based on a single period model of investment. The DM has to choose and allocate his available wealth among various securities. Commonly, some system constraints are assumed, one of which is that the proportion invested in each security is non-negative (Ogryczak, 2000). The mean variance methodology (Markowitz (1952)) for portfolio selection problem has been central to research activity and has served as a

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basis for the development of modern financial theory. In the literature, some algorithms such as those proposed by Sharpe (1963, 1967), and Elton et al. (1976) are generated in order to linearize and improve the efficiency calculation of the Markowitz covariance model (Nawrocki and Carter, 1998, and Shing and Nagasawa, 1999). The Markowitz model was generally criticized as not efficient with axiomatic models of preferences for choice under risk (Bell and Raiffa, 1988). Levy (1992) affirmed that models consistent with the preferences are based on the relation of stochastic dominance or on the expected utility theory. For that reason, Ballestero and Romero (1996), for example, suggested maximizing the investor expected utility of returns over the efficient frontier.

The expected return of a portfolio was used as an approximation because returns are random, and it is hardly possible that the investor can group a portfolio by attending to all of its possible returns (Liu, 1999). The Markowitz model extends the classic model by including one additional objective, variance, to describe risk. Arditti (1967) and Samuelson (1970) argue that higher moments cannot be neglected. Lai (1991) and Prakash et al. (2003) incorporated higher moments and particularly skewness in a polynomial goal programming. Portfolio selection problem can be then viewed as a multi-objective mathematical program.

In their paper, Steuer and Na (2003) presented a categorized bibliography on the application of multiple criteria decision making techniques. They noticed that 69% of the published papers used goal programming and multiple objective programming. Meanwhile, 29% dealt with portfolio selection problem. GP and CP have several applications in different fields such as portfolio selection problem which is usually characterized by several conflicting objectives. In GP and CP, DMs are able to establish, easily and precisely, goal values of the considered objectives. Lee and Chesser (1980), Levary and Avery (1984), and Kumar et al. (1978) proposed the use of GP for the portfolio selection problem, while Zeleny (1982) proposed the CP.

Zopounidis et al. (1999), grouped the main objectives considered when dealing with portfolio selection problems in three categories: (a) the corporal validity objectives, (b) the stocks acceptability by the investors, and (c) the financial objectives.

In many decision-making contexts, the DM is easily and precisely able to establish some parameters' values. However, such values in the portfolio selection problem are stochastic (Aouni et al., 2005). Stochastic programming (SP) and particularly multi-objective stochastic programming models can be used to deal with such difficulties (Ben Abdelaziz et al. (1995, 1999), Ben Abdelaziz and Mejri (2001), Ziemba and Mulvey (1998)). Several approaches have been proposed to solve the stochastic programming models such as the two-stage stochastic programming approach and the CCP approach developed by Charnes and Cooper (1959, 1963).

Among the applications of multi-objective stochastic programming in portfolio selection, (Ogryczak, 2000) extended Markowitz's model by developing a multi-criteria linear goal programming. In the model proposed by Shing and Nagasawa (1999) the mean and variance of return of securities have several scenarios with known probabilities. Ballestero (2001) proposed a formulation of stochastic goal programming (SGP) based on utility function and "Mean-Variance" model. Muhlemann et al. (1978) developed a multi-objective stochastic linear programming formulation of portfolio selection problem under uncertainty. Tamiz et al. (1996) proposed a two-stage goal programming model for portfolio selection. Aouni et al. (2005) explicitly introduced the DM's preferences and adapted CCP for the SGP model. They illustrated their formulation through a portfolio selection example where the goal values associated with each objective are considered normally distributed.

The aim of this paper is to present a model for portfolio selection in a context where some parameters are random and normally distributed. We propose a compromise chance constrained programming model (CCCP), which combine the CP model and the CCP approach. The CCCP allows DM to consider several conflicting and stochastic objectives. In this paper, we first present multi-objective models such as GP and CP and we introduce multi-objective CCP programming. Then, we present our CCCP model to multi-objective stochastic program. In order to illustrate the developed model, we test it on a sample of securities from the Tunisian stock exchange market.