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Continuous Optimization

New class of multiplicative algorithms for solving of entropy-linear programs

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Abstract

The general entropy-linear program (ELP) is considered. Two types of the new coordinate-wise multiplicative algorithms with p-active variables and feedback control with respect to dual variables and mixed type (dual and primal variables) are proposed for solving the problem. Study of algorithms convergence is based on a stability analysis of the auxiliary differential equations that are continuous analogues of the algorithms.

Keywords: Entropy maximization; Multiplicative algorithms; Row-action procedures; Active variables; Feedback control; G-convergence; Exponential Lagrange multipliers

1. Introduction

Entropy and the classical variational principle of the statistical physics are the effective tools for modelling and solving a lot of applied problems. There are many definitions of ''entropy'' functions. The book by Kapur (1989) contains some of them. The classical definition of the physical entropy was introduced by Boltzmann in 1871 (Boltzmann, 1871) and was developed by Fermi, Dirac and Bose, Einstein (Landau and Lifshitz, 1964). Introduced essentially later the information entropies by Shannon and Kulback and generalized information entropies by Fermi–Dirac and Bose–Einstein (Popkov, 1995) are closed to the definition of the physical entropy (they are Stirling-approximations of the Boltzmann-, or Fermi-, or Einstein-entropies).

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It is necessary to remark also that the notion of entropy as a measure of uncertainty is the base one although analytical expression of this measure can be different and it depends on the uncertainty model.

We will use the definitions of the generalized information entropies introduced in (Popkov, 1995). Such entropies and their particular forms are used as natural functionals for modelling transportation flows (Wilson, 1970, Imelbaev and Shmulyan, 1978) and stationary states of the macrosystems (Popkov, 1984, 1986), for urban and regional planning (Leonardy, 1978, Erlander, 1980, Batten and Roy, 1982). Entropy maximization are applied to image reconstruction from projections (Lent, 1977, Herman, 1980, 1982, Gull and Skilling, 1984, Byrne, 1993). A large number of applications of the entropy maximization principle is contained in (Kapur, 1989, Fang et al., 1997).

Entropy and entropy-like functions have some useful properties caused of their application in numerical procedures. In particular, one of directions in the theory of ill-posed problems is based on entropy regularization methods (Erlander, 1981, Smith and Grandy, 1985).

In this paper we consider the entropy maximization problem on polyhedral sets, which describe the most part of the above-named applied models. We will call such problem *the entropy-linear programs* (ELP).

Multiplicative procedures represent one of the classes of numerical procedures applied to ELP solving. Apparently, the first general approach for synthesis of such procedures was proposed in (Dubov et al., 1983). Simple multiplicative algorithm was applied to minimization of strictly convex functions on nonnegative orthant. Later, the multiplicative algorithms with respect to dual variables are used for solving conditional minimization and mathematical programming problems (Aliev et al., 1985, Popkov, 1988, 1995a). Also, the multiplicative algorithms are used for solving nonlinear equations (Popkov, 1996). The multiplicative procedures for finding nonnegative solutions of the minimization problems over nonnegative orthant were proposed again in the paper (Iusem et al., 1996).

Some types of the multiplicative algorithms may be derived from approach based on the Bregman function and generalized projections with Shannon's entropy. In this case we obtain so-called *row-action algorithms*, iterations of which have a multiplicative form. The first algorithm of this type was proposed in (Gordon et al., 1970) for three-dimensional image reconstruction from projections. Further, the row-action algorithms are developed and modified (Herman, 1982, Censor, 1981, Censor and Segman, 1987, Byrne, 1996, Censor and Zenios, 1997).

It is neccesary to note that in the most cited works the multiplicative algorithms are applied to the problems of entropy maximization with linear constraints equalities.

In this paper we consider the ELP problem, where a feasible set is described by the system of the equalities and inequalities. The regular procedure for design of multiplicative algorithms with p-active variables is proposed for this problem solving. On the basis of the procedure above we synthesize the algorithms with respect to dual variables and to mixed, dual and primal, variables simultaneously. The choice of the active variables is implemented by feedback control with respect to the current state of the iterative process. Convergence study of the multiplicative algorithms is based on the continuous analogues of the algorithms and equivalence of the iterative sequences generated by the dual and mixed type algorithms.

2. The ELP problem and optimality conditions

The ELP is a problem of mathematical programming with entropy goal-function and polyhedral feasible set. We will consider the ELP with two types of entropy goal-functions, namely:

$$H_B(y) = -\sum_{n=1}^{m} y_n \ln(y_n/a_n e), \tag{1}$$

where $e \cong 2.71$ and