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Original Article

Estimation of parameters for the exponentiated Pareto distribution based on progressively type-II right censored data

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Abstract In this paper, we derive the best linear unbiased estimates (BLEUs) and the maximum likelihood estimates (MLEs) of the location and scale parameters from the Exponentiated Pareto distribution based on progressively Type-II right censored order statistics. In addition, we use Monte-Carlo simulation method to obtain the mean square error of the best linear unbiased estimates and the maximum likelihood estimates and make comparison between them. Finally, we present numerical example.

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1. Introduction

Progressive censoring is very important in life-testing experiments. Its allowance for the removal of live-units from the experiment at various strange is an attractive feature as it will potentially save a lot for experimenter in terms of cost and time. In a series of papers, [1–5] discussed the inference problems for a wide range of distributions under this progressive censoring sampling scheme. These developments have been summarized by Cohen and Whitten [6], and more by Cohen [7]. While most of the above mentioned works were on the maximum likelihood method. Mann, Thomas and Wilson, and Cacciari, and Montanari [8–10] have discussed some linear inferences for the case of progressive Type-II right censoring. Mahmoud et al. [11] have derived approximate moments of progressively Type-II right censored order statistics from the Weibull Gamma distribution and using these moments to derive the best linear unbiased estimates and maximum likelihood estimates. Viveros

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and Balakrishnan [12] have proposed a conditional method of inference based on progressive Type II censored sample when the life time distributions are Weibull and exponential distributions. Aggarwala and Balakrishnan [13] have established an independence result for general progressive Type-II censored samples from the standard uniform distribution by generalizing the algorithm given by Balakrishnan and Sandhu [14]. This result is used in order to obtain moments of general progressive Type-II censored order statistics from the standard uniform distribution. Finally, they have derived the best linear unbiased estimators (BLUEs) of the parameters of one- and two-parameter uniform distributions. Fernandez [15] has discussed the problem of estimation of parameters of exponential distribution, on the basis of the general progressive Type-II censored sample. Balakrishnan and Sandhu [16] have derived the best linear unbiased estimators for the parameters of one- and two-parameters exponential distributions based on general progressive Type-II censored samples and the maximum likelihood estimators. Wu [17] has obtained the maximum likelihood estimates of the shape and scale parameters based on progressively Type-II censored sample from the Weibull distribution.

Recently [18,19] introduced a new distribution called generalized exponential distribution which has been studied quite extensively. In (2001), they discussed also a different method of estimations of the parameters of a generalized exponential distribution. Gupta et al. [20] showed that the exponentiated Pareto distribution can be used quite effectively in analyzing many lifetime data. M. M. Ali et al. [21,22] studied several exponentiated distributions including exponentiated Pareto distribution and discussed their properties. They showed that the exponentiated Pareto distribution gives a good fit to the tail-distribution of NASDAQ data. In (2010) they derived the distribution of the ratio of two independent exponentiated Pareto random variables X and Y and study its properties. Shawky and Hanaa Abu-Zinadah [23] studied how the different estimators of the unknown parameters of exponentiated Pareto distribution can behave for different sample sizes and for different parameter values.

In this paper, we derive the best linear unbiased estimates (BLUEs) and maximum likelihood estimates (MLE) of the location and scale parameters of progressively Type-II right censored data from exponentiated Pareto distribution .In addition, we use Monte-Carlo simulation method to make comparison of the MSE of BLUEs and MLE .

Let $X_1, X_2, \ldots, X_n$ denote a random sample from the Exponentiated Pareto distribution $EP(\theta,\alpha)$ with probability density function (pdf)

$$f(x_i) = \alpha\theta[1-(1+x_i)^{-\alpha}]^{\theta-1}(1+x_i)^{-\alpha-1}, \quad x_i \geq 0, \quad \alpha,\theta > 0, \tag{1.1}$$

and cumulative distribution function (cdf)

$$F(x_i) = [1-(1+x_i)^{-\alpha}]^{\theta}, \quad x_i \geq 0, \quad \alpha,\theta > 0. \tag{1.2}$$

2. Maximum likelihood estimation (MLE)

A maximum likelihood estimation is often the most feasible method to use when doing statistical inference, as the only information required to obtain MLEs is the joint distribution of the observed values. We are already well acquainted with the likelihood function to be maximized when a general progressively Type II censored sample based on n independent units with identical lifetime distributions from an arbitrary continuous distribution $F(x)$ is observed. Recall the likelihood function to be maximized will be

$$L(\mu,\sigma) = A(n, m-1)\prod_{i=1}^{m} f(x_i)[1-F(x_i)]^{R_i}, \tag{2.1}$$

where

$$A(n,m-1) = n(n-R_1-1)(n-R_1-R_2-2) \ldots (n-R_1-\cdots-R_{m-1}-m+1).$$

Let $X_{1:m:n}$, $X_{2:m:n}, \ldots, X_{m:m:n}$ be a progressively Type II right censored sample from exponentiated Pareto distribution, with censoring scheme $(R_1, R_2, \ldots, R_m)$ whose pdf. and cdf. given in Eqs. (1.1) and (1.2)

Then the likelihood function to be maximized for estimators of μ and σ (which we will denote by $\hat{\mu}$ and $\hat{\sigma}$)

$$L(\mu,\sigma) = (const.)(\alpha\theta)^m \prod_{i=1}^{m}\left[1-\left(1+\frac{x_i-\mu}{\sigma}\right)^{-\alpha}\right]^{\theta-1} \times\left(1+\frac{x_i-\mu}{\sigma}\right)^{-\alpha-1}\left[1-\left[1-\left(1+\frac{x_i-\mu}{\sigma}\right)^{-\alpha}\right]^{\theta}\right]^{R_i}.$$

For simplicity of notation, we will use X_i instead of $X_{i:m:n}$. The log-likelihood function may be then written as

$$\begin{aligned}\ln L(\mu,\sigma) = {} & const. + m\ln\alpha\theta \\ & +(\theta-1)\sum_{i=1}^{m}\ln\left[1-\left(1+\frac{x_i-\mu}{\sigma}\right)^{-\alpha}\right] \\ & -(\alpha+1)\sum_{i=1}^{m}\ln\left(1+\frac{x_i-\mu}{\sigma}\right) \\ & +\sum_{i=1}^{m}R_i\ln\left[1-\left[1-\left(1+\frac{x_i-\mu}{\sigma}\right)^{-\alpha}\right]^{\theta}\right],\end{aligned}$$

and hence we have the likelihood equations for μ and σ to be

$$\begin{aligned}&\frac{\alpha(\theta-1)}{\hat{\sigma}}\sum_{i=1}^{m}\frac{1}{\left[\left(1+\frac{x_i-\hat{\mu}}{\hat{\sigma}}\right)^{\alpha+1}-\left(1+\frac{x_i-\hat{\mu}}{\hat{\sigma}}\right)\right]} \\ &+\frac{(\alpha+1)}{\hat{\sigma}}\sum_{i=1}^{m}\frac{1}{\left(1+\frac{x_i-\hat{\mu}}{\hat{\sigma}}\right)}-\frac{\alpha\theta}{\hat{\sigma}}\sum_{i=1}^{m}R_i \\ &\times\frac{\left(1+\frac{x_i-\hat{\mu}}{\hat{\sigma}}\right)^{-\alpha-1}}{\left\{\left[1-\left(1+\frac{x_i-\hat{\mu}}{\hat{\sigma}}\right)^{-\alpha}\right]^{1-\theta}-\left[1-\left(1+\frac{x_i-\hat{\mu}}{\hat{\sigma}}\right)^{-\alpha}\right]\right\}}=0,\end{aligned} \tag{2.2}$$

and

$$\begin{aligned}&\frac{\alpha(\theta-1)}{\hat{\sigma}^2}\sum_{i=1}^{m}\frac{x_i-\hat{\mu}}{\left[\left(1+\frac{x_i-\hat{\mu}}{\hat{\sigma}}\right)^{\alpha+1}-\left(1+\frac{x_i-\hat{\mu}}{\hat{\sigma}}\right)\right]} \\ &+\frac{(\alpha+1)}{\hat{\sigma}^2}\sum_{i=1}^{m}\frac{x_i-\hat{\mu}}{\left(1+\frac{x_i-\hat{\mu}}{\hat{\sigma}}\right)}-\frac{\alpha\theta}{\hat{\sigma}^2}\sum_{i=1}^{m}R_i \\ &\times\frac{(x_i-\hat{\mu})\left(1+\frac{x_i-\hat{\mu}}{\hat{\sigma}}\right)^{-\alpha-1}}{\left\{\left[1-\left(1+\frac{x_i-\hat{\mu}}{\hat{\sigma}}\right)^{-\alpha}\right]^{1-\theta}-\left[1-\left(1+\frac{x_i-\hat{\mu}}{\hat{\sigma}}\right)^{-\alpha}\right]\right\}}=0.\end{aligned} \tag{2.3}$$