Journal of the Egyptian Mathematical Society (2016) 24, 479–486

Egyptian Mathematical Society

Journal of the Egyptian Mathematical Society

www.etms-eg.org
www.elsevier.com/locate/joems

Original Article

Dual solutions in hydromagnetic stagnation point flow and heat transfer towards a stretching/shrinking sheet with non-uniform heat source/sink and variable surface heat flux

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Received 30 July 2014; revised 27 June 2015; accepted 20 September 2015
Available online 18 November 2015

Keywords

Dual solutions;
Stagnation point flow;
Shrinking/stretching sheet;
Magnetic field;
Heat source/sink

Abstract The steady stagnation-point flow and heat transfer of a viscous, incompressible and heat generating/absorbing fluid over a shrinking sheet in the presence of a non-uniform heat source/sink is considered. The system of partial differential equations was transformed to a system of ordinary differential equations, which was solved numerically. Numerical results were obtained for the skin friction coefficient, the surface temperature as well as the velocity and temperature profiles for some values of the governing parameters. The study reveals that the range of velocity ratio parameter for which the solution exists increases as the magnetic field increase.

Math Subject Classification: 76N20-76R05-80A20

1. Introduction

Investigation on boundary-layer flow and heat transfer in a quiescent fluid driven by a continuous stretching sheet have been extensively investigated during the past decades owing to its importance in industrial and engineering applications. Examples are heat treatment of materials manufactured in an extrusion process and a casting process of materials. Cooling of stretching sheets is needed to assure the best quality of the material and requires dedicated control of the temperature and, therefore, knowledge of flow and heat transfer in such systems. Motivated by the process of polymer extrusion, in which the

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Peer review under responsibility of Egyptian Mathematical Society.

Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.joems.2015.09.004

Nomenclature

a, b, c	prescribed constants
A^*	space-dependent heat source/sink parameter
B^*	temperature-dependent heat source/sink parameter
B_0	magnetic induction
C_{fx}	local friction coefficient
c_p	specific heat at constant pressure
f	dimensionless stream velocity
k	thermal conductivity
M	magnetic parameter
Nu_x	local Nusselt number
Pr	Prandtl number
q'''	non-uniform heat source/sink
q_w	surface heat flux
Re_x	local Reynolds number
T	fluid temperature
T_∞	ambient temperature
u	velocity of the fluid in the x-direction
u_e	velocity of the ambient fluid
u_w	velocity of the stretching/shrinking sheet
v	velocity of the fluid in the y-direction
x, y	axial and normal coordinates

Greek symbols

η	similarity variable
θ	dimensionless temperature
λ	velocity ratio parameter
λ_c	critical value of velocity ratio parameter
μ	dynamic viscosity
ψ	stream function
ν	kinematic viscosity
ρ	fluid density
σ	electric conductivity

extrudate emerges from a narrow slit, Crane [1] was the first to give a similarity solution in a closed analytical form for the two-dimensional flow caused by a stretching plate. Subsequently; various aspects of the flow and heat transfer over a stretching surface have been examined by several investigators [2–7]. It is known that the properties of the final product depend greatly on the rate of cooling involved in manufacturing processes. It would be beneficial to have a controlled cooling system for these processes. An electrically conducting and heat generating/absorbing fluids seems to be a good candidate for some industrial applications such as in polymer technology and metallurgy because the flow can be regulated by external means through a magnetic field. The applied magnetic field may play an important role in controlling momentum and heat transfers in the boundary layer flow of different fluids over a stretching sheet. The use of magnetic fields has been also used in the process of purification of molten metals from non-metallic inclusions. Many works have been reported on flow and heat transfer of electrically conducting fluids over a stretched surface in the presence of magnetic field (see for instance, Chakrabarti and Gupta [8], Andersson [9], Chiam [10], Mahmoud [11], Abo-Eldahab and Abd El-Aziz [12,13], Abd El-Aziz and Salem [14] and Abd El-Aziz [15,16]). In recent times, the problem of flow and heat transfer over a shrinking sheet is relatively a new consideration in the laminar boundary layer flow. The surface velocity on the boundary towards a fixed point is known as a shrinking phenomenon. Shrinking sheet is a surface which decreases in size to a certain area due to an imposed suction or external heat. The flow induced by shrinking sheet exhibits quite distinct physical phenomena from the forward stretching flow. A search on the literature about this flow showed a few publications on the subject since it is quite a new type of flow. A steady boundary layer flow over a shrinking sheet is not possible as the vorticity generated in this case is not confined within the boundary layer. To maintain boundary layer structure the flow needs a certain amount of external suction at the porous sheet. Wang [17] first brought in the concept of the flow developed due to shrinking sheet while studying the behavior of liquid film on an unsteady stretching sheet. The existence and uniqueness of the similarity solution of the equation for the flow due to a shrinking sheet with suction were established by Miklavčič and Wang [18]. The flow induced by a shrinking sheet with constant or power-law velocity distribution was investigated recently by Fang [19] and Fang et al. [20].Wang [21] studied the stagnation flow towards a shrinking sheet and found that solutions do not exist for larger shrinking rates and may be non-unique in the two-dimensional case. The flow over an unsteady shrinking sheet was studied by Fang et al. [22] and the solution is an exact solution of the unsteady Navier–Stokes equations. Yacob et al. [23] investigated the heat transfer characteristics occurring during the melting process due to a stretching/shrinking sheet in a micropolar fluid. Bhattacharyya [24] analyzed the effects of partial slip on steady boundary layer stagnation-point flow of an incompressible fluid and heat transfer towards a shrinking sheet. The problem of a steady mixed convection flow on a moving plate in nanofluids has been investigated numerically by Subhashini and Sumathi [25]. Roşca and Pop [26] analyzed the problem of unsteady viscous flow over a curved stretching/shrinking surface with mass suction. The study of the boundary layer magnetohydrodynamic (MHD) flow towards a shrinking sheet has gained considerable attention of many researchers because of its frequent occurrence in industrial technology, geothermal applications, and high temperature plasmas applicable to nuclear fusion energy conversion, liquid metal fluids and MHD power generation systems. Shrinking problem can also be applied to study the capillary effects in smaller pores, the shrink-well behavior and the hydraulic properties of agricultural clay soils since associated changes in hydraulic and mechanical properties of such soils will seriously hamper predictions of the flow and transport processes which are essential for agricultural development and environmental management strategies. Also, shrinking film is one of the common applications of shrinking problems in industries. The shrinking film is very useful in packaging of bulk products since it can be unwrapped easily with adequate heat. Muhaimina et al. [27] studied the effect of chemical reaction, heat and mass transfer on nonlinear boundary layer past a porous shrinking sheet in the presence of suction. A series solution of three-dimensional MHD and rotating flow over a porous shrinking sheet was obtained by Hayat et al. [28]