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Original Article

Some studies of the interaction between N-two level atoms and three level atom

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Abstract In this paper, we present the analytical solution for the model that describes the interaction between a three level atom and two systems of N-two level atoms. The effect of the quantum numbers on the atomic inversion and the purity, for some special cases of the initial states, are investigated. We observe that the atomic inversion and the purity change remarkably by the change of the quantum numbers.

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1. Introduction

The use of statistical mechanics is fundamental to the concepts of quantum optics: light is described in terms of field operators for creation and annihilation of photons [1–7]. The development of quantum optics started in the 50s and 60s, motivated by the need to understand photodetection, statistics and coherence of light within the framework of quantum theory. In the late 50s, the seminal Hanbury-Brown and Twiss (HBT) experiment [8,9] demonstrated the bunching of photons emitted by a classical chaotic source and stimulated the development of the quantum theory of optical coherences by Glauber [10–12]. An important milestone in quantum optics was the demonstration of the non-classical character of fluorescence radiation emitted by a single atom and, later, the demonstration of photons sources showing photon antibunching [13–15]. In recent years the quantum optics of attenuating [16–22] and amplifying [23–27] dielectric media was developed, where optical modes are described as open quantum systems. Features of quantum optics are mainly based on three different types of interaction, namely, field–field, atom–atom, atom–field interaction. These interactions have been extensively considered in a huge number of papers; see for example Refs. [28–44]. Each one of these interactions represents a certain type of physical phenomena [45–50]. In fact these types of interactions have a strong relation between them so that one

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can transform one type to another [51]. This particular kind of the transformation would enable us to transform field–field interaction into atom–field interaction or atom–atom interaction. This depends on the nature of which kind of the Hamiltonian we use to carry out the transformation. The studying of any kind of these interaction would surely lead to more progress in the field of quantum optics and consequently it may open the door to observe or to report a new phenomenon in this field [52]. In the present communication we are concerned with the type of atom–atom (spin–spin) interaction, the interaction between a three level atom and two systems of N-two level atoms. The time evolution of dynamical systems has attracted considerable attention over the past several decades because of its various applications [53]. An important aspect in this regard is the atomic inversion and the purity. The atomic population inversion can be considered as one of the simplest important quantities, it is defined as "the difference between the probabilities of finding the atom in its exited state and in its ground state". The purity can range between zero, corresponding to a completely pure state, and $(1-\frac{1}{d})$, corresponding to a completely mixed state, (here, d is the dimension of the density matrix). Hence, we study the atomic inversion and the purity of a three level atom coupled to two systems of N-two level atoms as an application.

This paper is organized as follows: in Section 2, we will describe the Hamiltonian of the system of interest, and obtain the explicit analytical solution of the model describing the interaction between a three level atom and two systems of N-two level atoms. Different cases are studied to demonstrate the effects due to the quantum numbers m_1, m_2 on the atomic inversion $\langle S_z\rangle$ and the purity $P_S(t)$ of the model. Conclusions are summarized at the end of the paper.

2. The model

The Hamiltonian of our model describes the interaction between a three level atom coupled to two systems of N-two level atoms. In this case the Hamiltonian of the whole system can be written in the form:

$$H = H_0 + H_{int.}, \tag{1}$$

where

$$H_0 = \sum_{\alpha=1}^{\alpha=2} \omega_\alpha J_z^{(\alpha)} + \sum_{\beta=1}^{\beta=3} \Omega_\beta S_{\beta\beta}, \tag{2}$$

$$H_0 = \sum_{\gamma=1}^{\gamma=2} \omega_\gamma J_z^{(\gamma)} + \sum_{\beta=1}^{\beta=3} \Omega_\beta S_{\beta\beta}, \tag{3}$$

$$H_{int.} = \lambda_1\left[S_{21}J_+^{(1)} + J_-^{(1)}S_{12}\right] + \lambda_2\left[S_{32}J_+^{(2)} + J_-^{(2)}S_{23}\right], \tag{4}$$

$$J_L^{(\gamma)} = \frac{1}{2}\sum_{k_\gamma=1}^{N_\gamma} \sigma_L^{k_\gamma}, L = x, y, z, \tag{5}$$

$$\Omega_1 > \Omega_2 > \Omega_3, \tag{6}$$

$$J_\pm^{(\gamma)} = J_x^{(\gamma)} \pm iJ_y^{(\gamma)}, \tag{7}$$

where ω_γ, $\gamma = 1, 2$ are the strength of the field (the two systems of N-two level atoms). The operators S_{ij} satisfy the commutation relation

$$[S_{kl}, S_{nm}] = S_{km}\delta_{nl} - S_{nl}\delta_{km}, \tag{8}$$

$J_\pm^{(\alpha)}$ and $J_z^{(\alpha)}$ are the collective angular momentum operators for N-two level atoms, which satisfy the relations, while $J_\pm^{(\gamma)}$ and $J_z^{(\gamma)}$ are the collective angular momentum operators for N-two level atoms, which satisfy the relations

$$\left[J_+^{(\gamma)}, J_-^{(\beta)}\right] = 2J_z^{(\gamma)}\delta_{\gamma\beta}, \tag{9}$$

$$\left[J_z^{(\gamma)}, J_\pm^{(\beta)}\right] = \pm J_\pm^{(\gamma)}\delta_{\gamma\beta}, \tag{10}$$

with the operator $\sigma_L^{k_\gamma}$ are the usual Pauli matrices.

We define

$$|\Psi(0)\rangle = |\Psi(0)\rangle_{j,m}|\Psi(0)\rangle_s \tag{11}$$

$$|\Psi(0)\rangle_{j,m} = |j_1, m_1, j_2, m_2\rangle \tag{12}$$

$$|\Psi(0)\rangle_s = a(0)|1\rangle + b(0)|2\rangle + c(0)|3\rangle \tag{13}$$

$$|\,a(0)\,|^2 + |\,b(0)\,|^2 + |\,c(0)\,|^2 = 1 \tag{14}$$

Let

$$\begin{aligned}|\Psi(t)\rangle = {} & A_{m_1,m_2}(t)|1, j_1, m_1, j_2, m_2\rangle \\ & + B_{m_1,m_2}(t)|2, j_1, m_1+1, j_2, m_2\rangle \\ & + C_{m_1,m_2}(t)|3, j_1, m_1+1, j_2, m_2+1\rangle.\end{aligned} \tag{15}$$

From Schrödinger equation

$$i\frac{\partial}{\partial t}|\Psi(t)\rangle = H|\Psi(t)\rangle, \tag{16}$$

we get from Eqs. (1) and (15)

$$i\frac{dA_{m_1,m_2}(t)}{dt} = \alpha_1 A_{m_1,m_2}(t) + \Lambda_1(m_1)B_{m_1,m_2}(t), \tag{17}$$

$$\begin{aligned}i\frac{dB_{m_1,m_2}(t)}{dt} = {} & \alpha_2 B_{m_1,m_2}(t) + \Lambda_1(m_1)A_{m_1,m_2}(t) \\ & + \Lambda_2(m_2)C_{m_1,m_2}(t),\end{aligned} \tag{18}$$

$$i\frac{dC_{m_1,m_2}(t)}{dt} = \alpha_3 C_{m_1,m_2}(t) + \Lambda_2(m_2)B_{m_1,m_2}(t), \tag{19}$$

where

$$\alpha_1 = (\omega_1 m_1 + \omega_2 m_2 + \Omega_1), \tag{20}$$

$$\alpha_2 = \omega_1(m_1+1) + \omega_2 m_2 + \Omega_2, \tag{21}$$

$$\alpha_3 = \omega_1(m_1+1) + \omega_2(m_2+1) + \Omega_3, \tag{22}$$

$$\Lambda_k = \lambda_k\sqrt{(j_k - m_k)(j_k + m_k + 1)} \quad k = 1, 2, \tag{23}$$

where λ_k the coupling parameters between spins.

Define

$$\begin{aligned}& A_{m_1,m_2}(t) = a(t)e^{-i\alpha_1 t}, B_{m_1,m_2}(t) = b(t)e^{-i\alpha_2 t}, \\ & C_{m_1,m_2}(t) = c(t)e^{-i\alpha_3 t}\end{aligned} \tag{24}$$

by substituting from Eq. (24) in Eqs. (17)–(19) we get the following equations:

$$i\frac{da(t)}{dt}e^{-i\alpha_1 t} = \Lambda_1(m_1)b(t)e^{-i\alpha_2 t}, \tag{25}$$