Journal of the Egyptian Mathematical Society (2015) 23, 527–534

Egyptian Mathematical Society
Journal of the Egyptian Mathematical Society
www.etms-eg.org
www.elsevier.com/locate/joems

ORIGINAL ARTICLE

Generalized ψ^*-closed sets in bitopological spaces

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Received 25 January 2014; revised 4 December 2014; accepted 18 December 2014
Available online 2 February 2015

KEYWORDS
ij-ψ^*-closed sets;
ij-ψ^*-continuous functions;
$ij-T_{1/5}$ spaces;
$ij-T^{\psi^*}_{1/5}$ spaces;
$ij-{}^{\psi^*}T_{1/5}$ spaces

Abstract In this paper, we introduce and study a new class of sets in a bitopological space (X, τ_1, τ_2), namely, *ij*-ψ^*-closed sets, which settled properly in between the class of *ji*-α-closed sets and the class of *ij*-gα-closed sets. We also introduce and study new classes of spaces, namely, $ij-T_{1/5}$ spaces, *ij*-T_e spaces, *ij*-αT_e spaces, *ij*-T_l spaces and *ij*-αT_l spaces. As applications of *ij*-ψ^*-closed sets, we introduce and study four new classes of spaces, namely, $ij-T^{\psi^*}_{1/5}$ spaces, $ij-{}^{\psi^*}T_{1/5}$ spaces (both classes contain the class of $ij-T_{1/5}$ spaces), *ij*-αT_k spaces and *ij*-T_k spaces. The class of *ij*-T_k spaces is properly placed in between the class of *ij*-T_e spaces and the class of *ij*-T_l spaces. It is shown that dual of the class of $ij-T^{\psi^*}_{1/5}$ spaces to the class of *ij*-αT_e spaces is the class of *ij*-αT_k spaces and the dual of the class of $ij-{}^{\psi^*}T_{1/5}$ spaces to the class of $ij-T_{1/5}$ spaces is the class of $ij-T^{\psi^*}_{1/5}$ spaces and also that the dual of the class of *ij*-T_l spaces to the class of *ij*-T_k spaces is the class of *ij*-αT_k spaces. Further we introduce and study *ij*-ψ^*-continuous functions and *ij*-ψ^*-irresolute functions.

2010 MATHEMATICAL SUBJECT CLASSIFICATION: 54 C 55; 54 C 10; 54 C 10; 54 E 55

1. Introduction

Recently the topological structure τ on a set X has a lot of applications in many real life applications. The abstractness of a set X enlarges the range of its applications. For example, a special type of this structure is the basic structure for rough set theory [1]. Alexandroff topologies are widely applied in the field of digital topologies [2]. Moreover, τ and its generalizations are applied in biochemical studies [3].

The work presented in this paper will open the way for using two viewpoints in these applications. That is, to apply two topologies at the same time. The concepts of g-closed sets, gs-closed sets, sg-closed sets, gα-closed sets, αg-closed sets, gp-closed sets, gsp-closed sets and spg-closed sets have been introduced in topological spaces (cf. [4–10]). El-Tantawy and Abu-Donia [11] introduced the concepts of (*ij*-$GC(X)$, *ij*-$GSC(X)$, *ij*-$SGC(X)$, *ij*-$G\alpha C(X)$, *ij*-$\alpha GC(X)$, *ij*-$GPC(X)$, *ij*-$GSPC(X)$, and *ij*-$SPGC(X)$) subset of (X, τ_1, τ_2). Abd Allah and Nawar [12] introduced The concept of ψ^*-open sets and studied The properties of $T_{1/5}$, T_e, αT_e, T_l, αT_l. In this paper, we introduce a new class of sets in a bitopological space (X, τ_1, τ_2), namely, *ij*-ψ^*-closed sets, which settled properly in between the class of *ji*-α-closed sets and the class of *ij*-gα-closed sets. And we extend the properties to a bitopological space (X, τ_1, τ_2). Also we use

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Peer review under responsibility of Egyptian Mathematical Society.

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http://dx.doi.org/10.1016/j.joems.2014.12.005
1110-256X

the family of *ij*-ψ^*-closed sets to introduce some types of properties in (X, τ_1, τ_2), and we study the relation between these properties. The concepts of pre-continuous, semi-continuous, α-continuous, sp-continuous, g-continuous, αg-continuous, gα-continuous, gs-continuous, sg-continuous, gsp-continuous, spg-continuous, gp-continuous, gc-irresolute, gs-irresolute, αg-irresolute and gα-irresolute functions have been introduced in topological spaces (cf. [7,10,13–22]). El-Tantawy and Abu-Donia [11] introduced the concepts of (*ij-pre*-continuous, *ij*-semi-continuous, *ij*-α-continuous, *ij-sp*-continuous, *ij-g*-continuous, *ij-αg*-continuous, *ij-gα*-continuous, *ij-gs*-continuous, *ij-sg*-continuous, *ij-gsp*-continuous, *ij-spg*-continuous, *ij-gp*-continuous, *ij*-gc-irresolute, *ij-gs*-irresolute, *ij-αg*-irresolute and *ij-gα*-irresolute) functions in bitopological spaces. In this paper, we introduce a new functions in a bitopological space (X, τ_1, τ_2), namely, *ij*-ψ^*-continuous functions and *ij*-ψ^*-irresolute functions.

2. Preliminaries

Definition 2.1. [23] A subset A of a bitopological space (X, τ_1, τ_2) is called:

(1) *ij*-preopen if $A \subseteq \tau_i\text{-int}(\tau_j\text{-cl}(A))$ and *ij*-preclosed if $\tau_i\text{-cl}(\tau_j\text{-int}(A)) \subseteq A$.
(2) *ij*-semi-open if $A \subseteq \tau_j\text{-cl}(\tau_i\text{-int}(A))$ and *ij*-semi-closed if $\tau_j\text{-int}(\tau_i\text{-cl}(A)) \subseteq A$.
(3) *ij*-α-open if $A \subseteq \tau_i\text{-int}(\tau_j\text{-cl}(\tau_i\text{-int}(A)))$ and *ij*-α-closed if $\tau_i\text{-cl}(\tau_j\text{-int}(\tau_i\text{-cl}(A))) \subseteq A$.
(4) *ij*-semi-preopen if $A \subseteq \tau_j\text{-cl}(\tau_i\text{-int}(\tau_j\text{-cl}(A)))$ and *ij*-semi preclosed if $\tau_j\text{-int}(\tau_i\text{-cl}(\tau_j\text{-int}(A))) \subseteq A$.

The class of all *ij*-preopen (resp. *ij*-semi-open, *ij*-α-open and *ij*-semi-preopen) sets in a bitopological space (X, τ_1, τ_2) is denoted by *ij*-$PO(X)$ (resp. *ij*-$SO(X)$, *ij*-$\alpha O(X)$ and *ij*-$SPO(X)$). The class of all *ij*-preclosed (resp. *ij*-semi-closed, *ij*-α-closed and *ij*-semi-preclosed) sets in a bitopological space (X, τ_1, τ_2) is denoted by *ij*-$PC(X)$ (resp. *ij*-$SC(X)$, *ij*-$\alpha C(X)$ and *ij*-$SPC(X)$).

Definition 2.2. [23] For a subset A of a bitopological space (X, τ_1, τ_2), the *ij*-pre-closure (resp. *ij*-semi-closure, *ij*-α-closure and *ij*-semi-pre-closure) of A are denoted and defined as follow:

(1) $ij\text{-}pcl(A) = \cap\{F \subset X: F \in ij\text{-}PC(X), F \supseteq A\}$.
(2) $ij\text{-}scl(A) = \cap\{F \subset X: F \in ij\text{-}SC(X), F \supseteq A\}$.
(3) $ij\text{-}\alpha cl(A) = \cap\{F \subset X: F \in ij\text{-}\alpha C(X), F \supseteq A\}$.
(4) $ij\text{-}spcl(A) = \cap\{F \subset X: F \in ij\text{-}SPC(X), F \supseteq A\}$.

Dually, the *ij*-preinterior (resp. *ij*-semi-interior, *ij*-α-interior and *ij*-semi-preinterior) of A, denoted by *ij*-$pint(A)$ (resp. *ij*-$sint(A)$, *ij*-$\alpha int(A)$ and *ij*-$spint(A)$) is the union of all *ij*-preopen (resp. *ij*-semi-open, *ij*-α-open and *ij*-semi-preopen) subsets of X contained in A.

Definition 2.3. [11] A subset A of a bitopological space (X, τ_1, τ_2) is called:

(1) *ij-g*-closed (denoted by *ij*-$GC(X)$) if, $A \subseteq U$, $U \in \tau_i \Rightarrow j\text{-cl}(A) \subseteq U$.
(2) *ij-gs*-closed (denoted by *ij*-$GSC(X)$) if, $A \subseteq U$, $U \in \tau_i \Rightarrow ji\text{-scl}(A) \subseteq U$.
(3) *ij-sg*-closed (denoted by *ij*-$SGC(X)$) if, $A \subseteq U$, $U \in ij\text{-}SO(X) \Rightarrow ji\text{-scl}(A) \subseteq U$.
(4) *ij-gα*-closed (denoted by *ij*-$G\alpha C(X)$) if, $A \subseteq U$, $U \in ij\text{-}\alpha O(X) \Rightarrow ji\text{-}\alpha\text{cl}(A) \subseteq U$.
(5) *ij-αg*-closed (denoted by *ij*-$\alpha GC(X)$) if, $A \subseteq U$, $U \in \tau_i \Rightarrow ji\text{-}\alpha\text{cl}(A) \subseteq U$.
(6) *ij-gp*-closed (denoted by *ij*-$GPC(X)$) if, $A \subseteq U$, $U \in \tau_i \Rightarrow ji\text{-pcl}(A) \subseteq U$.
(7) *ij-gsp*-closed (denoted by *ij*-$GSPC(X)$) if, $A \subseteq U$, $U \in \tau_i \Rightarrow ji\text{-spcl}(A) \subseteq U$.
(8) *ij-spg*-closed (denoted by *ij*-$SPGC(X)$) if, $A \subseteq U$, $U \in ji\text{-}SPO(X)) \Rightarrow ji\text{-spcl}(A) \subseteq U$.

The complement of an *ij*-$GC(X)$ (resp. *ij*-$GSC(X)$, *ij*-$SGC(X)$, *ij*-$G\alpha C(X)$, *ij*-$\alpha GC(X)$, *ij*-$GPC(X)$, *ij*-$GSPC(X)$, and *ij*-$SPGC(X)$) subset of (X, τ_1, τ_2) is called an *ij*-$GO(X)$ (resp. *ij*-$GSO(X)$, *ij*-$SGO(X)$, *ij*-$G\alpha O(X)$, *ij*-$\alpha GO(X)$, *ij*-$GPO(X)$, *ij*-$GSPO(X)$, and *ij*-$SPGO(X)$) subset of (X, τ_1, τ_2).

Definition 2.4. [11] A function $f: (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called:

(1) *ij*-pre-continuous if $\forall\ V \in i\text{-}C(Y)$, $f^{-1}(V) \in ij\text{-}PC(X)$.
(2) *ij*-semi-continuous if $\forall\ V \in i\text{-}C(Y)$, $f^{-1}(V) \in ij\text{-}SC(X)$.
(3) *ij*-α-continuous if $\forall\ V \in i\text{-}C(Y)$, $f^{-1}(V) \in ij\text{-}\alpha C(X)$.
(4) *ij-sp*-continuous if $\forall\ V \in i\text{-}C(Y)$, $f^{-1}(V) \in ij\text{-}SPC(X)$.
(5) *ij-g*-continuous if $\forall\ V \in j\text{-}C(Y)$, $f^{-1}(V) \in ij\text{-}GC(X)$.
(6) *ij-αg*-continuous if $\forall\ V \in j\text{-}C(Y)$, $f^{-1}(V) \in ij\text{-}\alpha GC(X)$.
(7) *ij-gα-continuous* if $\forall\ V \in j\text{-}C(Y)$, $f^{-1}(V) \in ij\text{-}G\alpha C(X)$.
(8) *ij-gs*-continuous if $\forall\ V \in j\text{-}C(Y)$, $f^{-1}(V) \in ij\text{-}GSC(X)$.
(9) *ij-sg*-continuous if $\forall\ V \in j\text{-}C(Y)$, $f^{-1}(V) \in ij\text{-}SGC(X)$.
(10) *ij-gsp*-continuous if $\forall\ V \in j\text{-}C(Y)$, $f^{-1}(V) \in ij\text{-}GSPC(X)$.
(11) *ij-spg*-continuous if $\forall\ V \in j\text{-}C(Y)$, $f^{-1}(V) \in ij\text{-}SPGC(X)$.
(12) *ij-gp*-continuous if $\forall\ V \in j\text{-}C(Y)$, $f^{-1}(V) \in ij\text{-}GPC(X)$.
(13) *i*-continuous if $\forall\ V \in i\text{-}C(Y)$, $f^{-1}(V) \in i\text{-}C(X)$.
(14) *ij-gc*-irresolute if $\forall\ V \in ij\text{-}GC(Y)$, $f^{-1}(V) \in ij\text{-}GC(X)$.
(15) *ij-gs*-irresolute if $\forall\ V \in ij\text{-}GSC(Y)$, $f^{-1}(V) \in ij\text{-}GSC(X)$.
(16) *ij-αg*-irresolute if $\forall\ V \in ij\text{-}\alpha GC(Y)$, $f^{-1}(V) \in ij\text{-}\alpha GC(X)$.
(17) *ij-gα*-irresolute if $\forall\ V \in ij\text{-}G\alpha C(Y)$, $f^{-1}(V) \in ij\text{-}G\alpha C(X)$.

Definition 2.5. [12] A subset A of (X, τ) is called ψ^*-closed if $A \subseteq U$, $U \in G\alpha O(X) \Rightarrow \alpha\text{cl}(A) \subseteq U$. The complement of ψ^*-closed set is said to be ψ^*-open.

Definition 2.6. [12] A space (X, τ) is called:

(1) $T_{1/5}$ space if $G\alpha C(X) = \alpha C(X)$.
(2) $T^{\psi^*}_{1/5}$ space if $\psi^* C(X) = \alpha C(X)$.
(3) ${}^{\psi^*}T_{1/5}$ space if $G\alpha C(X) = \psi^* C(X)$.
(4) T_e space if $GSC(X) = \alpha C(X)$.
(5) αT_e space if $\alpha GC(X) = \alpha C(X)$.
(6) T_k space if $GSC(X) = \psi^* C(X)$.
(7) αT_k space if $\alpha GC(X) = \psi^* C(X)$.
(8) T_l space if $GSC(X) = G\alpha C(X)$.
(9) αT_l space if $\alpha GC(X) = G\alpha C(X)$.

Definition 2.7. [12] A function $f: (X, \tau) \to (Y, \sigma)$ is called:

(1) ψ^*-continuous if $\forall\ V \in C(Y)$, $f^{-1}(V) \in \psi^* C(X)$.
(2) ψ^*-irresolute if $\forall\ V \in \psi^* C(Y)$, $f^{-1}(V) \in \psi^* C(X)$.
(3) pre-ψ^*-closed if $A \in \psi^* C(X)$, $f(A) \in \psi^* C(Y)$.