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REVIEW PAPER

On generalizing covering approximation space

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Abstract In this paper, we present the covering rough sets based on neighborhoods by approximation operations as a new type of extended covering rough set models. In fact, we have introduced generalizations to W. Zhu approaches (Zhu, 2007). Based on the notion of neighborhood induced from any binary relation, four different pairs of dual approximation operators are defined with their properties being discussed. The relationships among these operators are investigated. Finally, an interesting theorem to generate different topologies is provided. Comparisons between these topologies are discussed. In addition, several examples and counter examples to indicate counter connections are investigated.

MATHEMATICS SUBJECT CLASSIFICATION: 54A05; 03E20; 97R20; 68U01; 68U35

Contents

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1. Introduction

In order to extract useful information hidden in voluminous data, many methods in addition to classical logic have been proposed. These include fuzzy set theory [2], rough set theory [3], computing with words [4–7] and computational theory for linguistic dynamic systems [8]. Rough set theory, proposed by Pawlak in the early 1980s [3,9], is a mathematical tool to deal with uncertainty and incomplete information. Since then we have witnessed a systematic, world-wide growth of interest in rough set theory [10–26] and its applications [27–34].

Nowadays, it turns out that this approach is of fundamental importance to artificial intelligence and cognitive sciences, especially in the areas of data mining, machine learning, decision analysis, knowledge management, expert systems, and pattern recognition. Rough set theory bears on the assumption that some elements of a universe may be indiscernible in view of the available information about the elements. Thus, the indiscernibility relation is the starting point of rough set theory. Such a relation was first described by equivalence relation in the way that two elements are related by the relation if and only if they are indiscernible from each other. In this framework, a rough set is a formal approximation of a subset of the universe in terms of a pair of unions of equivalence classes which give the lower and upper approximations of the subset. However, the requirement of equivalence relation as the indiscernibility relation is too restrictive for many applications. In other words, many practical data sets cannot be handled well by classical rough sets. In light of this, equivalence relation has been generalized to characteristic relation [35–37] similarity relation [38], tolerance relation [39–42], and even arbitrary binary relation [43–49] in some extensions of the classical rough sets. Another approach is the relaxation of the partition arising from equivalence relation to a covering. The covering of a universe is used to construct the lower and upper approximations of any subset of the universe [11,15,19,25,50]. In the literature, several different types of covering-based rough sets have been proposed and investigated; see, for example, [1,23,26,51–54] and the bibliographies therein. It is well-known that coverings are a fundamental concept in topological spaces and play an important role in the study of topological properties. This motivates the research of covering rough sets from the topology point of view. Some initial attempts have already been made along the way. For example, Zhu and Wang examined the topological properties of the lower and upper approximation operations for covering generalized rough sets in [34,55]. Wu et al. combined the notion of topological spaces into rough sets and then discussed the properties of topological rough spaces [56]. In [1], neighborhoods, another elementary concept in topology, have been used to define an upper approximation; some properties of approximation operations for this type of covering rough sets have been explored as well [1,24,52,57].

So, we can say that there are two directions (see Fig. 1.1) for generalizing rough set theory one of them is replacing the equivalence relation by an arbitrary binary relation such as Yao [58]; the other direction is replacing the partition arising from the equivalence relation to cover the universe such as Zakowski [45], Pomykala [28] and Willim Zhu [1]. But most of them had failed to achieve all the properties of original rough set theory and thus they put some conditions and restrictions.

Pawlak rough set model		**Generalized rough set model**
$\boxed{\text{equivalence relation}}$ (element based definition)	$G \longrightarrow$	$\boxed{\text{relation}}$ (element based definition)
$\Updownarrow$		$\Uparrow$
$\boxed{\text{partition}}$ (granule based definition)	$G \longrightarrow$	$\boxed{\text{covering}} \Rightarrow \boxed{\text{covering}}$ (granule based definition)
$\Updownarrow$		$\Downarrow$
$\boxed{\sigma\text{-algebra}}$ (subsystem based definition)	$G \longrightarrow$	$\boxed{\text{subsystem}}$ (subsystem based definition)

Figure 1.1 [59]: Schematic diagram of different formulations of approximation operators.

In the present paper, we introduce a framework for generalizing the two directions. In fact, we introduce the generalized covering approximation space "$\mathcal{G}_n - \boldsymbol{CAS}$" as a generalization to rough set theory and covering approximation space. Moreover, in our approaches $\mathcal{G}_n - \boldsymbol{CAS}$, four different approximations that satisfy all properties of original rough set theory without any conditions or restrictions are constructed.

Most real life situations need some sort of approximation to fit mathematical models. The beauty of using topology in approximation is achieved via obtaining approximation for qualitative concepts (i.e. subsets) without coding or using assumption. General topology is the appropriated mathematical model for every collection connected by relations. Relations were used in the construction of topological structures in several fields such as, structural analysis [60], general view of space time [61], biochemistry [62], biology [63], and rough set theory [3,9]. Recently, some topological concepts such as subbase, neighborhood and separation axioms have been applied to study covering-based rough sets. However, the topological space on covering-based rough sets and the corresponding topological properties on the topological covering-based rough space are not studied. This paper studies some of these problems. We introduce new method to generate different general topologies from any neighborhood space. The provided method can be considered an easy method to generate different topologies directly from the binary relation without using subbase or base. The used technique is useful in rough context or in covering-based rough sets since the concepts and the properties of generated topologies can be applied in rough set theory and covering-based rough set theory. We believe that the using of this method is easier in application field and it is useful for applying many topological concepts in future studies. This research not only can form the theoretical basis for further applications of topology on covering-based rough sets but also lead to the development of the rough set theory and artificial intelligence.

2. Basic concepts

In this section, we introduce the fundamental concepts which used through this paper.

Definition 2.1. "*Binary Relation*" [64]

Let A and B be sets, then a "*binary relation*" R from A to B (or between A and B) is a subset of a Cartesian product $A \times B$,