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ORIGINAL ARTICLE

Fully developed natural convective micropolar fluid flow in a vertical channel with slip



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Abstract The problem of fully developed natural convective micropolar fluid flow is investigated. The slip boundary conditions for fluid velocity are applied. Non-dimensional variables are introduced. The closed form solutions of the field equations are represented graphically. As expected, it can be seen that the increase in micropolarity parameter results in a decrease in the velocity and an increase in the microrotation. Also, it is observed that the increase in the slip parameter increases the velocity and decreases the microrotation. The no slip case can be recovered as a limiting case of this work when the slip parameter goes to infinity.

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1. Introduction

The theory of micropolar fluids is proposed by Eringen [1] to recover the inadequacy of Navier–Stokes theory to describe the correct behavior of some types of fluids with microstructure such as animal blood, muddy water, colloidal fluids, lubricants and chemical suspensions [1]. In the mathematical theory of micropolar fluids there is, in general, six degrees of freedom, three for translation and three for microrotation of

microelements. Extensive reviews of the theory and applications can be found in the review articles [2,3] and in the recent books [1,4].

Buoyancy forces, which results from density differences in a fluid caused by the temperature gradients, are responsible for the fluid motion in natural (or free) convection. Natural convection fluid flows play a significant role in many practical applications including, for example, cooling of electronic components. Natural convection of micropolar fluid flow has attracted the attention of many researchers because of its wide area of applications such as dilute suspensions of polymer fluids, liquid crystals, and chemical suspensions. Gorla et al studied the natural convection boundary layer flow of a micropolar fluid over a vertical plate with a uniform heat flux [5]. Chamkha et al. [6] considered the problem of fully developed free convection of a micropolar fluid in a vertical channel applying the classical no-slip boundary conditions. The free convective laminar flow in a vertical channel with one region

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filled with micropolar fluid and the other region with a viscous fluid assuming no-slip condition is investigated in [7]. Rohni et al. [8] discussed the problem of mixed convection boundary layer flow near the stagnation point on a heated permeable vertical surface embedded in a saturated porous medium with temperature slip effects.

In the last century, several studies have shown that the classical no-slip condition for velocity might not always hold and that fluid slippage might happen at the solid boundary [9–12]. In the literature, many researchers showed that the no-slip boundary condition may lead to singular or nonrealistic behavior (e.g. [13–15]). A general slip boundary condition that allows the possibility of fluid slip at a solid boundary has been proposed by Navier [16]. It states that the tangential relative velocity of the fluid at a point on the solid boundary is proportional to the tangential stress acting at that point. The constant of proportionality is called the slip coefficient and is assumed to depend only on the nature of the fluid and bounding surface [17]. Recently, the slip condition has been used extensively for viscous fluids [17–23], micropolar fluids [24,25], Maxwell fluids [26] and Ferrofluids [27].

Ahmadi [28] and others restricted gyro-viscosity parameter γ by assuming that $\gamma = (\mu + \kappa/2)j$. One may wonder about the effect of this assumption especially for steady fluid flows [1]. This restriction may be used when studying time dependent fluid flows to allow the field equations to predict the correct behavior of viscous fluid flows in the limiting case when micro-structure effects become negligible and microrotation reduces to the angular velocity [1,25]. This motivated the author to investigate the problem of fully developed free convective micropolar fluid flow in a vertical channel without taking this restriction into consideration. Moreover, the more realistic velocity slip boundary conditions are applied at the two vertical walls and its influence is studied.

2. Formulation of the problem

Let us consider the laminar free convection flow of an incompressible micropolar fluid between two vertical plates. The motion is assumed to be steady and fully developed and the walls are heated with different uniform temperatures. Working with the Cartesian coordinates (x, y, z) , with z axis normal to xy -plane, as shown in Fig. 1, the field equations governing the problem at hand will take the following forms [1].

Balance of momentum:

$$(\mu + \kappa) \frac{d^2 u}{dy^2} + \kappa \frac{dv}{dy} + \rho g \beta (T - T_0) = 0, \quad (1)$$

Balance of angular momentum:

$$\gamma \frac{d^2 v}{dy^2} - \kappa \frac{du}{dy} - 2\kappa v = 0, \quad (2)$$

Equation of energy:

$$\frac{d^2 T}{dy^2} = 0, \quad (3)$$

where $u(y)$ is the velocity component along x direction and $v(y)$ is the microrotation about z direction. Also, $T(y)$ represents the fluid temperature. (μ, κ) are the viscosity coefficients and γ is the gyro-viscosity parameter. ρ, β, g and T_0 are,

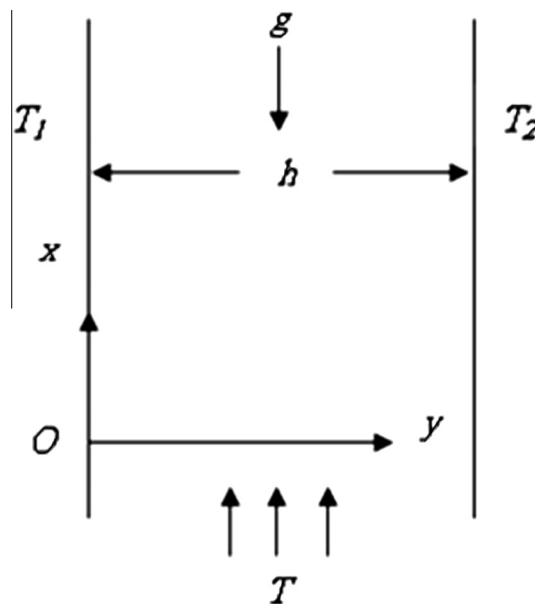


Fig. 1 Geometrical sketch of the problem.

respectively, fluid density, thermal expansion coefficient, gravitational acceleration and temperature of the plane surface.

The following slip and no-spin boundary conditions are proposed

$$\alpha u = \tau_{yx}, \quad T = T_1, \quad v = 0 \quad \text{on } y = 0, \quad (4)$$

$$\alpha u = \tau_{yx}, \quad T = T_2, \quad v = 0 \quad \text{on } y = h, \quad (5)$$

where T_1 and T_2 are the temperatures of the two walls $y = 0$ and $y = h$, respectively. $0 \leq \alpha \leq \infty$ is the slip parameter and h is the distance between the two walls. The shear stress τ_{yx} is given by [1]

$$\tau_{yx} = (\mu + \kappa) \frac{du}{dy} + \kappa v. \quad (6)$$

We now introduce the following non-dimensional variables

$$Y = \frac{y}{h}, \quad U = \frac{u}{U_0}, \quad N = \frac{h}{U_0} v, \quad \theta = \frac{T - T_0}{T_2 - T_0}, \quad T_{yx} = \frac{h}{\mu U_0} \tau_{yx}, \quad (7)$$

where $U_0 = \rho g \beta (T_2 - T_1) h^2 / \mu$.

3. Solution of the problem

Substituting the non-dimensional variables (7) into Eqs. (1)–(3), we get

$$(1 + K) \frac{d^2 U}{dY^2} + K \frac{dN}{dY} + \theta = 0, \quad (8)$$

$$\Gamma \frac{d^2 N}{dY^2} - K \frac{dU}{dY} - 2KN = 0, \quad (9)$$

$$\frac{d^2 \theta}{dY^2} = 0, \quad (10)$$

where $K = \kappa / \mu$ and $\Gamma = \gamma / \mu h^2$.

Note that, if we assumed that $\Gamma = 1 + K/2$, Eq. (9) returns to the classical form assumed by Ahmadi and others who restricted themselves to the relation $\gamma = (\mu + \kappa/2)j$.

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