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ORIGINAL ARTICLE

Two extensions of coupled coincidence point results for nonlinear contractive mappings

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Abstract In this paper, the notion of mixed f, g monotone mapping is introduced, and the coupled coincidence point theorem for nonlinear contractive mappings in partially ordered complete metric spaces has been proved. Presented theorems are generalizations of the recent fixed point theorems due to Lakshmikantham and Ćirić (2009) [17] and include several recent developments. Also, using the theory of countable extension of t-norm, it has been proved that a common fixed point theorem given in Ćirić (2011) [12] hold for a more general classes of t-norms in fuzzy metric spaces. Their theorem can be used to investigate a large class of problems and has discussed the existence and uniqueness of solution for a periodic boundary value problem.

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1. Introduction and preliminaries

The Banach contraction principle is one of the most important fixed point theorem, end generalized in various directions. For more results, we refer [1–25]. Boyd and Wong [4] extended the Banach contraction principle to the case of nonlinear contraction mappings. Ran and Reurings [23] proved a Banach contraction principle in partially ordered metric spaces. After that, many authors have continued research (see [2,3,17,20,21]). In a recent papers, Bhaskar and Lakshmikantham [3] and Lakshmikantham and Ćirić [17] proved a coupled fixed point results for mixed monotone and contraction mapping in partially ordered metric spaces. Bhaskar and Lakshmikantham [3] noted that their theorem can be used to investigate a large class of problems and has discussed the existence and uniqueness of solution for a periodic boundary value problem.

Definition 1.1. Let $(X, \leqslant)$ be a partially ordered set and $F : X \to X$ is such that for $x, y \in X, x \leqslant y$ implies $F(x) \leqslant F(y)$.

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Then, a mapping F is said to be non-decreasing. Similarly, it is defined as a non-increasing mapping.

Lakshmikantham and Ćirić [17] introduced the following notions of a mixed monotone mapping and a coupled fixed point.

Definition 1.2 [17]. Let $(X, \leqslant)$ be a partially ordered set, $F : X \times X \to X$ and $g : X \to X$. We say F has the mixed g-monotone property if F is monotone g-non-decreasing in its first argument and is monotone g-non-increasing in its second argument, that is, for any $x, y \in X$,

$$x_1, x_2 \in X, \quad g(x_1) \leqslant g(x_2) \text{ implies } F(x_1, y) \leqslant F(x_2, y) \tag{1}$$

and

$$y_1, y_2 \in X, \quad g(y_1) \leqslant g(y_2) \text{ implies } F(x, y_1) \geqslant F(x, y_2). \tag{2}$$

Definition 1.3 [17]. An element $(x, y) \in X \times X$ is called a coupled coincidence point of a mapping $F : X \times X \to X$ and $g : X \to X$ if

$$F(x, y) = g(x), \quad F(y, x) = g(y).$$

Definition 1.4 [17]. Let X be a non-empty set and $F : X \times X \to X$ and $g : X \to X$. We say F and g are commutative if

$$g(F(x, y)) = F(g(x), g(y))$$

for all $x, y \in X$.

The main theoretical results of Lakshmikantham and Ćirić in [17] are the following coupled coincidence point theorems.

Theorem 1.5 [17]. *Let $(X, \leqslant)$ be a partially ordered set and suppose there is a metric d on X such that (X, d) is a complete metric space. Assume there is a function $\varphi : [0, +\infty) \to [0, +\infty)$ with $\varphi(t) < t$ and $\lim_{r \to t+} \varphi(r) < t$ for each $t > 0$ and also suppose $F : X \times X \to X$ and $g : X \to X$ are such that F has the mixed g-monotone property and*

$$d(F(x, y), F(u, v)) \leqslant \varphi\left(\frac{d(g(x), g(u)) + d(g(y), g(v))}{2}\right) \tag{3}$$

for all $x, y, u, v \in X$ for which $g(x) \leqslant g(u)$ and $g(y) \geqslant g(v)$. Suppose $F(X \times X) \subseteq g(X)$, g is continuous and commutes with F and also suppose either

(a) *F is continuous or*
(b) *X has the following property:*

$$(i) \text{ if a non-decreasing sequence } \{x_n\} \to x, \text{ then } x_n \leqslant x \text{ for all } n, \tag{4}$$

$$(ii) \text{ if a non-increasing sequence } \{y_n\} \to y, \text{ then } y \leqslant y_n \text{ for all } n. \tag{5}$$

If there exists $x_0, y_0 \in X$ such that

$$g(x_0) \leqslant F(x_0, y_0) \text{ and } g(y_0) \geqslant F(y_0, x_0),$$

then there exist $x, y \in X$ such that

$$g(x) = F(x, y) \text{ and } g(y) = F(y, x),$$

that is, F and g have a coupled coincidence.

Recently, coupled coincidence point results can see in [26–29]. Inspired with Definition 1.3 we introduce in this paper the concept of a mixed fg-monotone mapping and prove a coupled coincidence fixed point theorems for nonlinear contractive mappings in partially ordered complete metric spaces.

Since the probabilistic metric spaces introduced by Menger [30] are a natural generalization of a metric spaces, Ćirić et al. [12,13] introduced a concept of monotone-generalized contraction in partially ordered probabilistic metric space, and they proved a common fixed point theorem. In [12] Ćirić et al. introduced the concept of mixed monotone-generalized contraction in partially ordered probabilistic metric space, and they proved a coupled coincidence and coupled fixed point theorem where they used a t-norm of H-type. Inspired with that in this paper, we proved that the result in [12] hold for a more general class of t-norms.

Through this paper with Δ^+, we denoted the space of all distribution function, i.e. $\Delta^+ = \{F : \mathbb{R} \cup [0, 1] \to [0, 1] : F$ is left continuous and non-decreasing on $\mathbb{R}$, $F(0) = 0$ and $F(+\infty) = 1\}$ and the subset $D^+ \subseteq \Delta^+$ is the set $D^+ = \{F \in \Delta^+ : l^- F(+\infty) = 1\}$, where the $l^- f(x)$ denotes the left limit of the function f at the point x. The space Δ^+ is partially ordered by the usual point-wise ordering of function, i.e. $F \leqslant G$ if and only if $F(x) \leqslant G(x)$ for all $x \in \mathbb{R}$. The maximal element for Δ^+ in this order is the distribution function

$$\varepsilon_0(t) = \begin{cases} 0, & \text{if } t \leqslant 0 \\ 1, & \text{if } t > 0. \end{cases}$$

Definition 1.6. A mapping $T : [0, 1] \times [0, 1] \to [0, 1]$ is called a triangular norm (a t-norm) if the following conditions are satisfied:

- $T(a, 1) = a$ for all $a \in [0, 1]$;
- $T(a, b) = T(b, a)$ for all $a, b \in [0, 1]$;
- $a \geqslant b,\ c \geqslant d \Rightarrow \quad T(a, c) \geqslant T(b, d) \quad (a, b, c, d \in [0, 1])$;
- $T(a, T(b, c)) = T(T(a, b), c) \quad (a, b, c \in [0, 1])$.

The following are the four basic t-norms (see [31]):

$$T_M(x, y) = \min(x, y), \quad T_P(x, y) = x \cdot y,$$
$$T_L(x, y) = \max(x + y - 1, 0)$$
$$T_D(x, y) = \begin{cases} \min(x, y) & \text{if} \max(x, y) = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Some important families of t-norms are given in the following example (see [31]):

Example 1.7.

(i) The Dombi family of t-norms $(T_\lambda^D)_{\lambda \in [0,\ \infty]}$, which is defined by

$$T_\lambda^D(x,\ y) = \begin{cases} T_D(x,\ y), & \lambda = 0 \\ T_M(x,\ y), & \lambda = \infty \\ \frac{1}{1 + \left(\left(\frac{1-x}{x}\right)^\lambda + \left(\frac{1-y}{y}\right)^\lambda\right)^{1/\lambda}}, & \lambda \in (0,\ \infty). \end{cases}$$