Journal of the Egyptian Mathematical Society (2015) 23, 410–415

Egyptian Mathematical Society

Journal of the Egyptian Mathematical Society

www.etms-eg.org
www.elsevier.com/locate/joems

ORIGINAL ARTICLE

Investigation of heat transfer in flow of Burgers' fluid during a melting process

M. Awais [a,*], T. Hayat [b,c], A. Alsaedi [c]

[a] Department of Mathematics, COMSATS Institute of Information Technology, Attock 43600, Pakistan
[b] Department of Mathematics, Quaid-i-Azam University, 45320, Islamabad 44000, Pakistan
[c] Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box 80257, Jeddah 21589, Saudi Arabia

Received 8 May 2013; revised 5 December 2013; accepted 12 April 2014
Available online 28 May 2014

KEYWORDS

Burgers' fluid;
Stagnation point flow;
Melting process;
Nonlinear computations

Abstract The heat transfer analysis during melting process in steady flow of an incompressible Burgers' fluid over stretching surface is investigated. The two-dimensional flow equations are modeled and then simplified by employing boundary layer analysis. The solution to the arising nonlinear problem is computed. Interpretation of various emerging parameters is given through graphs for velocity and temperature fields. Furthermore tables are constructed in order to show a comparative study with the previous published results. Comparison shows an excellent agreement with the previous limiting investigations in the field.

MATHEMATICS SUBJECT CLASSIFICATION: 76BXX; 76WXX; 76MXX

1. Introduction

In the recent years, researchers have keen interest in the flow of non-Newtonian fluid due to their practical applications in the field of engineering and technology. For-instance in designing plunge bearings and radial diffusers, in thermal oil recovery, cooling of strips, in traffic engineering where traffic is assumed as continuous fluid etc. The non-Newtonian fluids in view of diverse characteristics cannot be described by a single constitutive relation. Hence researchers have proposed various non-Newtonian fluid models to predict different rheological features. The survey of literature witnesses that there is replete literature on the topic concerning the flows of differential type fluids (a subclass of non-Newtonian fluids) in boundary layer region whereas such flows are in scarce for the rate type fluid models. It is because of the fact that the derivation of governing equations for the rate type fluids in two-dimensional flow analysis is much more complex than those of differential type fluids. However recent researchers have paid considerable attention on rate type fluids. For-instance Jamil et al. [1] examined the helical flows of Oldroyd-B fluids. Constantly accelerated flow between two sided walls perpendicular to the plate has been investigated by Fetecau et al. [2]. Pahlavan

* Corresponding author. Tel.: +92 51 90642172.
E-mail address: awais_mm@yahoo.com (M. Awais).

Peer review under responsibility of Egyptian Mathematical Society.

ELSEVIER Production and hosting by Elsevier

1110-256X © 2014 Production and hosting by Elsevier B.V. on behalf of Egyptian Mathematical Society.
http://dx.doi.org/10.1016/j.joems.2014.04.004

and Sadeghy [3] examined the homotopy analysis method for solving unsteady MHD flow of Maxwellian fluids above impulsively stretching sheets. Hayat et al. [4] investigated the effects of mass transfer on the stagnation point flow of an upper-convected Maxwell (UCM) fluid. Sajid et al. [5] presented boundary layer flow of an Oldroyd-B fluid in a region of stagnation point over a stretching sheet. Similar solution for the three-dimensional flow of an Oldroyd-B fluid has been presented by Hayat et al. [6]. Recently Hayat et al. [7] investigated stagnation point flow of Burgers' fluid over a stretching surface.

Melting heat transport phenomenon has been introduced recently in view of its relevance to some particular engineering problem. For-instance in the magma solidification, melting of permafrost, preparations of semi-conductor materials, etc. The seminal work by Epstein and Cho [8] incorporated melting effects on heat transport phenomenon to submerged bodies. The work of Epstein has been extended by the various researchers. Cheng and Lin [9] studied melting effect on mixed convective heat transfer with aiding and opposing external flows from the vertical plate in a liquid-saturated porous medium. Ishak et al. [10] studied melting heat transfer in steady laminar flow over a moving surface. Melting heat transfer in boundary layer stagnation-point flow toward a stretching/shrinking sheet has been analyzed by Bachok et al. [11]. Hayat et al. [12] investigated melting heat transfer in the stagnation-point flow of a second grade fluid. Royon and Guiffant [13] analyzed the heat transfer properties of slurry of stabilized paraffin during a melting process.

The purpose of current study is to analyze the characteristics of melting heat transfer on the boundary layer flow of Burgers' fluid [7] over a stretched surface. Nonlinear analysis is formulated. The solutions are derived by homotopy analysis method (HAM) which has been already applied to provide series solutions of various nonlinear problems [14–18]. Graphical results for dimensionless velocity and temperature are displayed and discussed. The numerical values of local Nusselt number have been obtained for various values of embedded parameters. Several tables are constructed in order to make a comparative study with various published articles in the limiting sense. These tables assure the validity of the present investigation.

2. Mathematical analysis

Consider the stagnation point flow of Burgers' fluid. The flow is induced by the stretching of sheet coinciding with the plane $y = 0$ whereas fluid occupies the region $y \geqslant 0$. The x- and y-axes are chosen along and perpendicular to sheet respectively. Velocity of stagnation point flow is taken as $U_e(x) = ax$ and the velocity of stretching sheet is $U_w(x) = cx$ where a and c are the positive constants. Further, the effect of melting heat transfer is taken into account. It is assumed that the temperature of the melting surface is T_m while the temperature in the free stream is T_∞, where $T_\infty > T_m$. The boundary layer equations governing the flow in the absence of viscous dissipation effects are modeled as follows

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$\begin{aligned} & u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + \lambda_1\left[u^2\frac{\partial^2 u}{\partial x^2} + v^2\frac{\partial^2 u}{\partial y^2} + 2uv\frac{\partial^2 u}{\partial x\partial y}\right] \\ & + \lambda_2\left[u^3\frac{\partial^3 u}{\partial x^3} + v^3\frac{\partial^3 u}{\partial y^3} + u^2\left(\frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial x^2} + 2\frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial x\partial y}\right)\right. \\ & +3v^2\left(\frac{\partial v}{\partial y}\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right) + 3uv\left(u\frac{\partial^3 u}{\partial x^2\partial y} + v\frac{\partial^3 u}{\partial x\partial y^2}\right) \\ & \left.+2uv\left(2\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial x^2} + \frac{\partial v}{\partial x}\frac{\partial^2 u}{\partial y^2} + \frac{\partial v}{\partial y}\frac{\partial^2 u}{\partial x\partial y}\right)\right] = \nu\frac{\partial^2 u}{\partial y^2} + U_e\frac{dU_e}{dx} \\ & +\nu\lambda_3\left[u\frac{\partial^3 u}{\partial x\partial y^2} + v\frac{\partial^3 u}{\partial y^3} - \frac{\partial u}{\partial x}\frac{\partial^2 u}{\partial y^2} - \frac{\partial u}{\partial y}\frac{\partial^2 v}{\partial y^2}\right], \end{aligned} \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} \tag{3}$$

with the corresponding boundary conditions given by

$$\begin{aligned} & u = U_w(x) = cx, \quad v = 0, \quad T = T_m \quad \text{at } y = 0 \\ & u \to U_e(x) = ax, \quad \frac{\partial u}{\partial y} \to 0, \quad T \to T_\infty \quad \text{as } y \to \infty, \\ & k\left(\frac{\partial T}{\partial y}\right)_{y=0} = \rho[\lambda + c_s(T_m - T_0)]v(x,0), \end{aligned} \tag{4}$$

The boundary conditions for heat transport phenomena state that the heat conducted to the melting surface is equal to the heat of melting plus the sensible heat required to raise the temperature of the solid T_0 to its melting temperature T_m (see Epstein and Cho [11]) Moreover in above equations u and v denote the velocity components in the x- and y-directions respectively, λ_1 and λ_2 the relaxation times respectively, λ_3 the retardation time, ν the kinematic viscosity, T the fluid temperature, T_m the mean fluid temperature, α the thermal diffusivity of the fluid, k the thermal conductivity, λ the latent heat of the fluid, c_s the heat capacity of the solid surface, c_p the specific heat and k the thermal conductivity, The velocity components in terms of stream function ψ and similarity transformations can be expressed as

$$u = \frac{\partial\psi}{\partial y}, \quad v = -\frac{\partial\psi}{\partial x}, \tag{5}$$

$$\psi = x\sqrt{c\nu}f(\eta), \quad \theta(\eta) = \frac{T - T_m}{T_\infty - T_m}, \quad \eta = \sqrt{\frac{c}{\nu}}y. \tag{6}$$

Now Eq. (1) is satisfied automatically and Eqs. (2) and (3) become

$$\begin{aligned} & f''' - f'^2 + ff'' + \beta_1(2ff'f'' - f^2f''') + \beta_2(f^3f'''' - 2ff'^2f'' - 3f^2f''^2) \\ & + \beta_3(f''^2 - ff'''') + A^2 = 0, \end{aligned} \tag{7}$$

$$\theta'' + \Pr f\theta' = 0, \tag{8}$$

$$\begin{aligned} & f'(0) = 1, \quad \Pr f(0) + M_e\theta'(0) = 0, \quad \theta(0) = 0, \\ & f'(\infty) = A, \quad f''(\infty) = 0, \quad \theta(\infty) = 1, \end{aligned} \tag{9}$$

in which β_1 and β_2 denote the Deborah numbers in terms of relaxation times respectively, β_3 the Deborah number in terms of retardation times, A the stagnation point parameter, Pr the Prandtl number and Me the dimensionless melting parameter. These are defined as follows: