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ORIGINAL ARTICLE

Modified alternative (G'/G) -expansion method to general Sawada–Kotera equation of fifth-order



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Abstract The alternative (G'/G) -expansion method has been further modified by introducing the generalized Riccati equation to construct new exact solutions. In order to illustrate the novelty and advantages of this approach, the general Sawada–Kotera (GSK) equation is considered and abundant new exact traveling wave solutions are obtained in a uniform way. These solutions may be imperative and significant for the explanation of some practical physical phenomena. It is shown that the modified alternative (G'/G) -expansion method is an efficient and advance mathematical tool for solving nonlinear partial differential equations in mathematical physics.

MATHEMATICS SUBJECT CLASSIFICATION: C02; C30; C32

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1. Introduction

The rapid development of nonlinear sciences witnesses a wide range of reliable and efficient techniques (see for example [1–43]) which are of great help in tackling physical problems even of highly complex nature. After the observation of soliton phenomena by John Scott Russell in 1834 [1,25] and since the KdV equation was solved by Gardner et al. [2] by inverse scattering method, finding exact solutions of nonlinear evolution equations (NLEEs) has turned out to be one of the most exciting and particularly active areas of research. The appearance of solitary wave solutions in nature is quite common. Bell-shaped sech-solutions and kink-shaped tanh-solutions

model wave phenomena in elastic media, plasmas, solid state physics, condensed matter physics, electrical circuits, optical fibers, chemical kinematics, fluids, bio-genetics, etc. The traveling wave solutions of the KdV equation and the Boussinesq equation which describe water waves are well-known examples. Apart from their physical relevance, the closed-form solutions of NLEEs if available facilitate the numerical solvers in comparison, and aids in the stability analysis. In soliton theory, there are several techniques to deal with the problems of solitary wave solutions for NLEEs, such as, Hirota's bilinear transformation [3], Backlund transformation [4], improved homotopy perturbation [5], Darboux transformation [6], Tanh-function [7], homogeneous balance [8], Jacobi elliptic function [9,10], F-expansion [11] and Exp-function [12–15].

Recently, Wang et al. [16] established a widely used direct and concise method called the (G'/G) -expansion method for obtaining the exact travelling wave solutions of NLEEs, where $G(\xi)$ satisfies the second order linear ordinary differential equation (ODE) $G'' + \lambda G' + \mu G = 0$, where λ and μ are arbitrary constants. Applications of the (G'/G) -expansion method can be found in the articles [17–26] for better understanding.

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In order to establish the effectiveness and reliability of the (G'/G)-expansion method and to expand the possibility of its application, further research has been carried out by several researchers. For instance, Zhang et al. [27] presented an improved (G'/G)-expansion method to seek more general traveling wave solutions. Zayed [28] presented a new approach of the (G'/G)-expansion method where $G(\xi)$ satisfies the Jacobi elliptic equation $[G'(\xi)]^2 = e_2G^4(\xi) + e_1G^2(\xi) + e_0$, e_2, e_1, e_0 are arbitrary constants, and obtained new exact solutions. Zayed [29] again presented an alternative approach of this method in which $G(\xi)$ satisfies the Riccati equation $G'(\xi) = A + BG^2(\xi)$, where A and B are arbitrary constants. Still, substantial work has to be done in order for the (G'/G)-expansion method to be well established, since every nonlinear equation has its own physically significant rich structure. For finding the new exact solutions of NLEEs, it is important to present various method and ansatz, but it seems to be more important how to obtain more new exact solutions to NLEEs under the known method and ansatz. In the present article, we further modify the alternative (G'/G)-expansion method (presented by Zayed [29]) by introducing the generalized Riccati equation mapping, its twenty seven solutions and constructed abundant new traveling wave solutions of the general Sawada–Kotera equation which is a special case of the fifth-order KdV equation (fKdV)

$$u_t + u_{xxxxx} + \gamma uu_{xxx} + \beta u_x u_{xx} + \alpha u^2 u_x = 0, \tag{1.1}$$

where α , β and γ are arbitrary nonzero and real parameters, and $u = u(x, t)$ is a differentiable function. The fKdV Eq. (1.1) is an important mathematical model with wide applications in quantum mechanics and nonlinear optics, furthermore describes motions of long waves in shallow water under gravity and in a one-dimensional nonlinear lattice. The general SK equation is characterized by the values

$$\beta = \gamma, \quad \alpha = \frac{1}{5}\gamma^2. \tag{1.2}$$

With (1.2), (1.1) reduces to general SK equation:

$$u_t + u_{xxxxx} + \gamma uu_{xxx} + \gamma u_x u_{xx} + \frac{1}{5}\gamma^2 u^2 u_x = 0. \tag{1.3}$$

The article is arranged as follows: In Section 2, the modified alternative (G'/G)-expansion method is discussed. In Section 3, we apply this method to the nonlinear fifth-order KdV equation (fKdV) pointed out above; in Section 4, graphical representation and in Section 5 conclusions are given.

2. Methodology

Consider the general nonlinear PDE of the type

$$P(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}, \dots) = 0, \tag{2.1}$$

where $u = u(x, t)$ is an unknown function, P is a polynomial in $u(x, t)$ and its partial derivatives in which the highest order partial derivatives and the nonlinear terms are involved. The main steps of the modified alternative (G'/G)-expansion method combined with the generalized Riccati equation mapping are as follows:

Step 1: The travelling wave variable ansatz

$$u(x, t) = u(\xi), \quad \xi = x - Vt, \tag{2.2}$$

where V is the speed of the traveling wave, permits us to transform Eq. (2.1) into an ODE:

$$Q(u, u', u'', u''', \dots) = 0, \tag{2.3}$$

where the superscripts stands for the ordinary derivatives with respect to ξ .

Step 2: If possible, integrate Eq. (2.3) term by term one or more times. This yields constant(s) of integration.

Step 3: Suppose the traveling wave solution of Eq. (2.3) can be expressed by a polynomial in (G'/G) as follows:

$$u(\xi) = \sum_{n=0}^m a_n \left(\frac{G'}{G}\right)^n, \quad a_m \neq 0 \tag{2.4}$$

where $G = G(\xi)$ satisfies the generalized Riccati equation,

$$G' = r + pG + qG^2, \tag{2.5}$$

where a_n ($n = 0, 1, 2, \dots, m$), r , p and q are arbitrary constants to be determined later. The generalized Riccati Eq. (2.5) has twenty seven solutions [41] as follows:

Family 1. When $p^2 - 4qr < 0$ and $pq \neq 0$ (or $rq \neq 0$), the solutions of Eq. (2.5) are,

$$G_1 = \frac{1}{2q} \left[-p + \sqrt{4qr - p^2} \tan\left(\frac{1}{2} \sqrt{4qr - p^2} \xi\right) \right],$$

$$G_2 = -\frac{1}{2q} \left[p + \sqrt{4qr - p^2} \cot\left(\frac{1}{2} \sqrt{4qr - p^2} \xi\right) \right],$$

$$G_3 = \frac{1}{2q} \left[-p + \sqrt{4qr - p^2} \left(\tan\left(\sqrt{4qr - p^2} \xi\right) \pm \sec\left(\sqrt{4qr - p^2} \xi\right) \right) \right],$$

$$G_4 = -\frac{1}{2q} \left[p + \sqrt{4qr - p^2} \left(\cot\left(\sqrt{4qr - p^2} \xi\right) \pm \csc\left(\sqrt{4qr - p^2} \xi\right) \right) \right],$$

$$G_5 = \frac{1}{4q} \left[-2p + \sqrt{4qr - p^2} \left(\tan\left(\frac{1}{4} \sqrt{4qr - p^2} \xi\right) - \cot\left(\frac{1}{4} \sqrt{4qr - p^2} \xi\right) \right) \right],$$

$$G_6 = \frac{1}{2q} \left[-p + \frac{\sqrt{(A^2 - B^2)(4qr - p^2)} - A\sqrt{4qr - p^2} \cos\left(\sqrt{4qr - p^2} \xi\right)}{A \sin\left(\sqrt{4qr - p^2} \xi\right) + B} \right],$$

$$G_7 = \frac{1}{2q} \left[-p + \frac{\sqrt{(A^2 - B^2)(4qr - p^2)} + A\sqrt{4qr - p^2} \cos\left(\sqrt{4qr - p^2} \xi\right)}{A \sin\left(\sqrt{4qr - p^2} \xi\right) + B} \right],$$

where A and B are two non-zero real constants and satisfy the condition $A^2 - B^2 > 0$.

$$G_8 = \frac{-2r \cos\left(\frac{1}{2} \sqrt{4qr - p^2} \xi\right)}{\sqrt{4qr - p^2} \sin\left(\frac{1}{2} \sqrt{4qr - p^2} \xi\right) + p \cos\left(\frac{1}{2} \sqrt{4qr - p^2} \xi\right)},$$

$$G_9 = \frac{2r \sin\left(\frac{1}{2} \sqrt{4qr - p^2} \xi\right)}{-p \sin\left(\frac{1}{2} \sqrt{4qr - p^2} \xi\right) + \sqrt{(4qr - p^2)} \cos\left(\frac{1}{2} \sqrt{4qr - p^2} \xi\right)},$$

$$G_{10} = \frac{-2r \cos\left(\sqrt{4qr - p^2} \xi\right)}{\sqrt{(4qr - p^2)} \sin\left(\sqrt{4qr - p^2} \xi\right) + p \cos\left(\sqrt{4qr - p^2} \xi\right) \pm \sqrt{(4qr - p^2)}}$$

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