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ORIGINAL ARTICLE

Numerical simulation of nanofluid flow with convective boundary condition

Kalidas Das [a,*], Pinaki Ranjan Duari [b], Prabir Kumar Kundu [b]

[a] *Dept. of Mathematics, Kalyani Govt. Engg. College, Kalyani 741235, West Bengal, India*
[b] *Dept. of Mathematics, Jadavpur University, Kolkata 700032, West Bengal, India*

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Abstract In this paper, the heat and mass transfer of an electrically conducting incompressible nanofluid over a heated stretching sheet with convective boundary condition is investigated. The transport model includes the effect of Brownian motion with thermophoresis in the presence of thermal radiation, chemical reaction and magnetic field. Lie group transformations are applied to the governing equations. The transformed ordinary differential equations are solved numerically by employing Runge–Kutta–Fehlberg method with shooting technique. Numerical results for temperature and concentration profiles as well as wall heat and mass flux are elucidated through graphs and tables.

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1. Introduction

In the recent past a new class of fluids, namely nanofluids has attracted the attention of the science and engineering community because of the many possible industrial applications of these fluids. An innovative way of improving the thermal conductivities of heat transfer fluids is to suspend small solid particles in the fluids. Nanofluids are nanometer-sized particles (diameter less than 50 nm) dispersed in a base fluid such as water, ethylene glycol, toluene and oil. Addition of high thermal conductivity metallic nanoparticles (e.g., aluminum, copper, silicon, silver and titanium or their oxides) increases the thermal conductivity of such mixtures; thus enhancing their overall energy transport capability. The enhancement of thermal conductivities by nanofluids was first discussed by Choi [1]. It should be noticed that there have been published several recent papers [2–5] on the mathematical and numerical modeling of convective heat transfer in nanofluids. The boundary layer flow of a nanofluid caused by a stretching surface has drawn the attention of a growing number of researchers [6–10] because of its immense potential to be used as a technological tool in many engineering applications.

The effect of radiation on heat transfer problems has studied by Makinde [11], Ibrahim et al. [12], Hayat et al. [13], Das [14] and Nadeem et al. [15]. Lie group analysis, also

* Corresponding author. Tel.: +91 9748603199.
E-mail addresses: kd_kgec@rediffmail.com (K. Das), pinakiranjanduari@gmail.com (P.R. Duari), kunduprabir@yahoo.co.in (P.K. Kundu).

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known as symmetry analysis, is the most powerful, sophisticated, and systematic method for finding similarity solution of non-linear differential equations and is widely used in non-linear dynamical system, especially in the range of deterministic chaos. This technique has been applied by many researchers [16–19] to study different flow phenomena over different geometrics arising in fluid mechanics, chemical engineering and other engineering branches. Hamad and Ferdows [20] considered similarity solution of boundary layer stagnation-point flow toward a heated porous stretching sheet saturated with a nanofluid using Lie group analysis. Recently, heat transfer problems for boundary layer flow concerning with a convective boundary condition were investigated by Ishak [21], Makinde and Aziz [22]. Recently, radiation effects on MHD nanofluid flow toward a stretching surface with convective boundary condition were discussed by Akbar et al. [23].

The aim of the present work was to study the effects of the thermal radiation on the heat and mass transfer of an electrically conducting incompressible nanofluid over a heated stretching sheet with convective boundary conditions. The flow is permeated by a uniform transverse magnetic field in presence of Brownian motion, chemical reaction with thermophoresis.

2. Mathematical analysis

The steady two-dimensional boundary layer flow of an electrically conducting nanofluid over a heated stretching sheet is considered in the region $y > 0$. Keeping the origin fixed, two equal and opposite forces are applied along the x-axis which results in stretching of the sheet and a uniform magnetic field of strength B_0 is imposed along the y-axis. It is assumed that the velocity of the external flow is $U(x) = ax$ and the velocity of the stretching sheet is $u_w(x) = bx$ where a is a positive constant and b is a positive (stretching sheet) constant. The chemical reaction and thermal radiation is taking place in the flow.

Under the above conditions, the governing boundary layer equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + v\frac{\partial^2 u}{\partial y^2} - \frac{v}{k}(u-U) - \frac{\sigma B_0^2}{\rho}(u-U), \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\kappa}{(\rho c_p)_f}\frac{\partial^2 T}{\partial y^2} + \frac{Q_0}{(\rho c_p)_f}(T - T_\infty) + \tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + \frac{D_T}{T_\infty}\left(\frac{\partial T}{\partial y}\right)^2\right] - \frac{1}{(\rho c_p)_f}\frac{\partial q_r}{\partial y} \tag{3}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + \frac{D_T}{T_\infty}\frac{\partial^2 T}{\partial y^2} - k_1(C - C_\infty) \tag{4}$$

where u, v are the velocity components along the x and y-axis respectively, T is temperature, k is the permeability of the porous medium, v is the kinematic viscosity, σ is the electrical conductivity, C_p is the specific heat at constant pressure, $\tau = (\rho c)_p/(\rho c)_f$ is the ratio of the effective heat capacity of the nanoparticle material and the base fluid, ρ_f is the density of base fluid, ρ_p is the nanoparticle density, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, k_1 is the rate of chemical reaction.

The radiative heat flux term q_r by using the Rosseland approximation is given by

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y} \tag{5}$$

where σ^* is the Stefan–Boltzmann constant and k^* is the mean absorption coefficient. Assuming that the differences in temperature within the flow are such that T^4 can be expressed as a linear combination of the temperature, T^4 may be expanded in Taylor's series about T_∞ and neglecting higher order terms, one may get

$$T^4 = 4T_\infty^3 T - 3T_\infty^4 \tag{6}$$

Thus

$$\frac{\partial q_r}{\partial y} = -\frac{16T_\infty^3\sigma^*}{3k^*}\frac{\partial^2 T}{\partial y^2} \tag{7}$$

The boundary conditions at the plate surface and far into the cold fluid may be written as

$$\left.\begin{array}{ll} u = u_w(x), v = v_w, -\kappa\frac{\partial T}{\partial y} = h_w(T_f - T_w), C = C_w & \text{for } y = 0, \\ u \to U(x), T \to T_\infty, C \to C_\infty & \text{as } y \to \infty \end{array}\right\} \tag{8}$$

where v_w is the wall mass transfer velocity and T_f is the convective fluid temperature.

Introducing the following non-dimensional variables:

$$x' = \frac{x}{\sqrt{vb}}, \quad y' = \frac{y}{\sqrt{vb}}, \quad u' = \frac{u}{\sqrt{vb}}, \quad v' = \frac{v}{\sqrt{vb}}, \quad U' = \frac{U}{\sqrt{vb}}, \quad \theta = \frac{T - T_\infty}{T_f - T_\infty}, \quad \phi = \frac{C - C_\infty}{C_w - C_\infty}, \tag{9}$$

and using classical Lie group approach along the same lines as in Das [10] and Hamad and Ferdows [20], we get

$$\eta = y, \quad \psi = xf(\eta), \quad \theta = \theta(\eta), \quad \phi = \phi(\eta) \tag{10}$$

Substituting (10) into Eqs. (2)–(4) we finally obtain the following system of non-linear ordinary differential equations

$$f''' + ff'' - f'^2 + \frac{a^2}{b^2} - \left(M^2 + \frac{1}{K}\right)\left(f' - \frac{a}{b}\right) = 0, \tag{11}$$

$$(1 + Nr)\frac{1}{Pr}\theta'' + f\theta' + \lambda\theta + Nb\theta'\phi' + Nt\theta'^2 = 0 \tag{12}$$

$$\phi'' + LePrf\phi' + \frac{Nt}{Nb}\theta'' - Kr\phi = 0 \tag{13}$$

The corresponding boundary conditions (8) become

$$\left.\begin{array}{ll} f = S, f' = 1, \theta' = -\gamma(1-\theta), \phi = 1 & \text{at } \eta = 0 \\ f' \to \frac{a}{b}, \theta \to 0, \phi \to 0 & \text{as } \eta \to \infty \end{array}\right\} \tag{14}$$

where $Pr = \frac{v}{\alpha}$ is the Prandtl number, $Le = \frac{\alpha}{D_B}$ is the Lewis number, $Nr = \frac{4T_\infty^3\sigma^*}{3k^*\kappa}$ is the thermal parameter, $Nb = \frac{\tau D_B(C_w - C_\infty)}{v}$ is the Brownian motion parameter, $Nt = \frac{\tau D_T(T_w - T_\infty)}{vT_\infty}$ is the thermophoresis parameter, $S = \frac{v_w}{\sqrt{bv}}$ is the suction/injection parameter, $Kr = \frac{k_1 v}{bD_B}$ is the chemical reaction rate parameter, $K = \frac{bk}{v}$ is the permeability parameter, $M = B_0\sqrt{\frac{\sigma}{b\rho}}$ is the magnetic field parameter and $\gamma = \frac{h_w\sqrt{vb}}{\kappa}$ is the surface convection parameter.

The quantities of physical interest in this problem are the local Nusselt number Nu and the local Sherwood number Su which are defined as

$$Nur = Re_x^{-1/2}Nu = -(1 + Nr)\theta'(0), \tag{15}$$

$$Shr = Re_x^{-1/2}Sh = -\phi'(0) \tag{16}$$