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ORIGINAL ARTICLE

Heat transfer analysis for squeezing flow between parallel disks

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Abstract Heat transfer analysis for the squeezing magneto-hydrodynamic (MHD) flow of a viscous incompressible fluid between parallel disks is considered. Upper disk is movable in upward and downward directions while the lower disk is fixed but permeable. Viable similarity transforms are used to convert the conservation law equations to a system of nonlinear ordinary differential equations. Resulting system is solved by using variational iteration method (VIM). Influence of flow parameters is discussed and numerical solution is sought using RK-4 method. A convergent solution is obtained just after few numbers of iterations.

MATHEMATICS SUBJECT CLASSIFICATION: 35Q79; 76D05; 76M55

1. Introduction

Heat transfer in rapidly moving engines and machines with lubricants inside has been an active field of research. For safe and consistent working of such machines it is necessary to study heat transfer in these systems. Several attempts are reported in this regard after the pioneer work done by Stefan [1]. Two-dimensional MHD squeezing flow between parallel plates has been examined by Siddiqui et al. [2]. For parallel disk similar problem has been discussed by Domairy and Aziz [3]. Both used homotopy perturbation method (HPM) to determine the solution.

Joneidi et al. [4] studied the mass transfer effect on squeezing flow between parallel disks using homotopy analysis method (HAM). Most recently, influence of heat transfer in the MHD squeezing Flow between parallel disks has been investigated by Hayat et al. [5]. They used HAM to solve the resulting nonlinear system of ordinary differential equations.

Motivated by the preceding work here we present heat transfer analysis for the MHD squeezing flow between parallel disks. Well known variational iteration method (VIM) [7–11] has been employed to solve system of highly nonlinear differential equations that govern the flow. VIM is a strong

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analytical technique and has been employed by several researchers in recent times to study different type of problems [12–21]. The main positive features of this technique is its simplicity, selection of initial approximation, compatibility with the nonlinearity of physical problems of diversified complex nature, minimal application of integral operator and rapid convergence [21].

Numerical solution is also sought to check the validity of analytical solution. A detailed comparison between purely analytical solution obtained by VIM and the numerical solution obtained by employing RK-4 method is presented. It is evident from this article that the VIM provides excellent results with less amount of laborious computational work.

2. Mathematical formulation

MHD flow of a viscous incompressible fluid is taken into consideration through a system consisting of two parallel infinite disks distance $h(t) = H(1 - at)^{1/2}$ apart. Magnetic field proportional to $B_0(1 - at)^{1/2}$ is applied normal to the disks. It is assumed that there is no induced magnetic field. T_w and T_h represent the constant temperatures at $z = 0$ and $z = h(t)$ respectively. Upper disk at $z = h(t)$ is moving with velocity $\frac{aH(1-at)^{-1/2}}{2}$ toward or away from the static lower but permeable disk at $z = 0$ as shown in Fig. 1. We have chosen the cylindrical coordinates system (r, ϕ, z). Rotational symmetry of the flow $(\partial/\partial\phi = 0)$ allows us to take azimuthal component v of the velocity $V = (u, v, w)$ equal to zero. As a result, the governing equation for unsteady two-dimensional flow and heat transfer of a viscous fluid can be written as [6]

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{1}$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z}\right) = -\frac{\partial \hat{p}}{\partial r} + \mu\left(\frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial z^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2}\right) - \frac{\sigma}{\rho}B^2(t)u, \tag{2}$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z}\right) = -\frac{\partial \hat{p}}{\partial z} + \mu\left(\frac{\partial^2 w}{\partial r^2} + \frac{\partial^2 w}{\partial z^2} + \frac{1}{r}\frac{\partial w}{\partial r}\right), \tag{3}$$

$$C_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + w\frac{\partial T}{\partial z}\right) = \frac{K_0}{\rho}\left(\frac{\partial^2 T}{\partial r^2} + \frac{\partial^2 T}{\partial z^2} + \frac{1}{r}\frac{\partial T}{\partial r} - \frac{u}{r^2}\right) + v\left\{2\frac{u^2}{r^2} + \left(\frac{\partial u}{\partial z}\right)^2 + 2\left(\frac{\partial w}{\partial z}\right)^2 + 2\left(\frac{\partial u}{\partial r}\right)^2 + 2\left(\frac{\partial w}{\partial r}\right)^2 + 2\frac{\partial u}{\partial z}\frac{\partial w}{\partial r}\right\}, \tag{4}$$

Auxiliary conditions are [5]

$$u = 0, \quad w = \tfrac{dh}{dt} \quad \text{at } z = h(t)$$
$$u = 0, \quad w = -w_0 \quad \text{at } z = 0. \tag{5}$$

$$T = T_w \quad \text{at } z = 0$$
$$u = T_h \quad \text{at } z = h(t). \tag{6}$$

u and w here are the velocity components in r and z directions respectively, μ is dynamic viscosity, $\hat{p}$ is the pressure and ρ is density. Further T denotes temperature, K_0 is thermal conductivity, C_p is specific heat, v is kinematic viscosity and w_0 is suction/injection velocity.

Using the following transformations [5]

$$u = \frac{ar}{2(1-at)}f'(\eta), \quad w = -\frac{aH}{\sqrt{1-at}}f(\eta),$$
$$B(t) = \frac{B_0}{\sqrt{1-at}}, \quad \eta = \frac{z}{H\sqrt{1-at}}, \quad \theta = \frac{T - T_h}{T_w - T_h}, \tag{7}$$

into Eqs. (2)–(4) and eliminating pressure terms from the resulting equations, we obtain

$$f'''' - S(\eta f''' + 3f'' - 2ff''') - M^2 f'' = 0, \tag{8}$$

$$\theta'' + S\,\Pr(2f\theta' - \eta\theta') - \Pr\,Ec\left(f''^2 + 12\delta^2 f'^2\right) = 0, \tag{9}$$

with the associated conditions

$$f(0) = A, \quad f'(0) = 0, \quad \theta(0) = 1,$$
$$f(1) = \frac{1}{2}, \quad f'(1) = 0, \quad \theta(1) = 0, \tag{10}$$

where S denotes the squeeze number, A is suction/injection parameter, M is Hartman number, Pr Prandtl number, Ec modified Eckert number, and δ denotes the dimensionless length defined as

$$S = \frac{aH^2}{2v}, \quad M^2 = \frac{aB_0^2 H^2}{v}, \quad \Pr = \frac{\mu C_p}{K_0},$$
$$Ec = \frac{1}{C_p(T_w - T_h)}\left(\frac{ar}{2(1-at)}\right)^2, \quad \delta^2 = \frac{H^2(1-at)}{r^2}. \tag{11}$$

Skin friction coefficient and the Nusselt number are defined in terms of variables (7) as

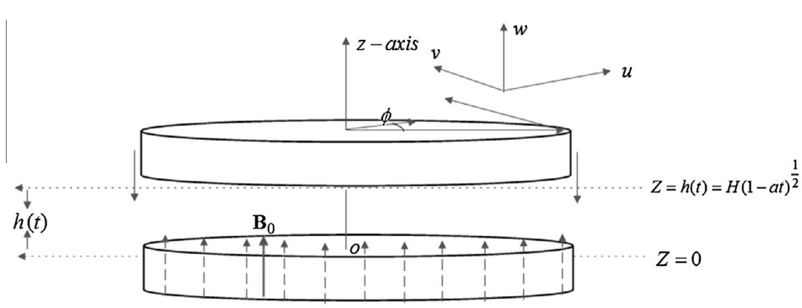

Figure 1 Geometry of the problem.