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ORIGINAL ARTICLE

Nanofluid flow over a non-linear permeable stretching sheet with partial slip

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Abstract In the present study, the problem of boundary layer flow of a nanofluid over non-linear permeable stretching sheet at prescribed surface temperature in the presence of partial slip is investigated numerically. By means of proper similarity variables, the governing equations are transformed to ordinary differential equations which are solved using symbolic software MATHEMATICA. The similarity solutions that depend on slip parameter, stretching parameter, etc. are elucidated through graphs and tables.

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1. Introduction

The flow over a stretching sheet is relevant to several important engineering applications in the field of metallurgy and chemical engineering processes. These applications involve the cooling of continuous strips or filaments by drawing them through a quiescent fluid. The steady two dimensional boundary layer flow of Newtonian fluid over a stretching surface has been studied by Crane [1]. After this pioneering work the flow field over a stretching surface has drawn considerable attention and a good amount of literature has been generated on this problem [2–5]. In this study the fluid velocity is assumed to be zero relative to the solid boundary. But this is not true for fluid flows at the micro- and nanoscale. Investigation shows that slip flow happens when the characteristic size of the flow system is small or the flow pressure is very low. To describe the phenomenon of slip, Navier [6] introduced a boundary condition which states that the component of the fluid velocity tangential to the boundary walls is proportional to tangential stress. Martin and Boyd [7] analyzed Blasius boundary layer problem in the presence of slip boundary condition. The hydrodynamic flow in the presence of partial slip over a stretching sheet with suction has been studied by Wang [8]. Das [9] analyzed the slip effects on heat and mass transfer in MHD micropolar fluid flow. Recently, Das [10] investigated convective heat transfer of nanofluids over a stretching sheet in the presence of partial slip and thermal radiation.

However, all these studies are restricted to linear stretching of the sheet. It is worth mentioning that the stretching is not necessarily linear, as in a polymer extrusion process. The problem of non-linear stretching sheet for different cases of fluid flow has also been analyzed by different researchers. Gupta and Gupta [11] first point out in their study that the stretching

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of the sheet may not necessarily be linear. In view of this, Vajravelu [12] studied flow and heat transfer over a non-linear stretching sheet. Cortell [13] extended the model proposed by [12] considering two different types of thermal boundary conditions on the sheet, constant surface temperature and prescribed surface temperature. Prasad et al. [14] investigated the mixed convection heat transfer over a non-linear stretching surface with variable fluid properties. Recently, Yazdi et al. [15] discussed the slip flow and heat transfer over a non-linear permeable stretching surface.

A nanofluid is a new class of heat transfer fluids that contain a base fluid and nanoparticles. Nanofluids have been shown to increase the thermal conductivity and convective heat transfer performance of the base liquids. One of the possible mechanisms for anomalous increase in the thermal conductivity of nanofluids is the Brownian motions of the nanoparticles inside the base fluids. It should be noticed that there have been published several recent papers [16,17] on the mathematical and numerical modeling of natural convection heat transfer in nanofluids. A comprehensive survey of convective transport in nanofluids was made by Buongiorno [18] and Kakac and Pramuanjaroenkij [19]. The Buongiorno model [18] has also been used by Khan and Pop [20] to study the boundary layer flow of a nanofluid past a stretching sheet. The boundary layer flow of a nanofluid caused by a stretching surface has drawn the attention of many researchers [21–23]. Very recently Rana and Bhargava [24] investigated the boundary layer flow of a nanofluid flow over a non-linearly stretching sheet.

There have been many theoretical models developed to describe slip flow along the surface. However, to the best of my knowledge, no investigation has been made yet to analyze the slip flow and heat transfer of a nanofluid past a non-linear stretching permeable surface at prescribed surface temperature. The objective of present article was therefore to extend the work of [24] by taking steady boundary layer flow and heat transfer of a nanofluid in the presence of partial slip over a non-linear permeable stretching surface at prescribed surface temperature.

2. Mathematical formulation

Consider the boundary layer flow of nanofluid over a non-linear permeable stretching surface. The flow takes place at $y \geqslant 0$, where y is the coordinate measured normal to the stretching surface. The flow is generated, due to the stretching of the sheet that emerges out of a slit at $x = 0,\ y = 0$. Let us assume that the speed at a point on the plate is proportional to the power of its distance from the slit and the boundary layer approximation are applicable. The sheet is assumed to vary non-linearly with distance x from the leading edge i.e.,

$$u_w = ax^n \tag{1}$$

where a is a positive constant and n is non-linear stretching parameter. The stretching surface is maintained at prescribed surface temperature, T_w as follows:

$$T = T_w(= T_\infty + bx^r) \quad \text{at} \quad y = 0 \tag{2}$$

where b is a positive constant, r is the surface temperature parameter in the prescribed surface temperature boundary condition and T_∞ is the temperature of the fluid far away from the surface. Special case of constant surface temperature is obtained by introducing r equal to zero.

The governing boundary layer equations for this investigation are as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{3}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \tag{4}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} + \tau\left[D_B\frac{\partial C}{\partial y}\frac{\partial T}{\partial y} + (D_T/T_\infty)\left(\frac{\partial T}{\partial y}\right)^2\right] \tag{5}$$

$$u\frac{\partial C}{\partial x} + v\frac{\partial C}{\partial y} = D_B\frac{\partial^2 C}{\partial y^2} + (D_T/T_\infty)\frac{\partial^2 C}{\partial y^2} \tag{6}$$

The associated boundary conditions are

$$\left.\begin{array}{l} u = u_w + u_s,\ v = \pm v_w,\ T = T_w,\ C = C_w \quad \text{at} \quad y = 0 \\ u \to 0,\ T \to T_\infty,\ C \to C_\infty \quad \text{as} \quad y \to \infty \end{array}\right\} \tag{7}$$

where u, v are the velocity components along x and y-axis respectively, ν is the kinematic viscosity, α is the thermal diffusivity, $\tau = (\rho c)_p/(\rho c)_f$ is the ratio between the effective heat capacity of the nanoparticle material and heat capacity of the fluid, C is the nanoparticle volumetric fraction, ρ_p is the density of the particles, ρ_f is the density of the base fluid, D_B is the Brownian diffusion coefficient, D_T is the thermophoretic diffusion coefficient, v_w is the suction/injection and u_s is the slip velocity which is assumed to be proportional to the local wall stress as follows:

$$u_s = l\left.\frac{\partial u}{\partial y}\right|_{y=0} \tag{8}$$

where l is the slip length as a proportional constant of the slip velocity.

By using similarity transformations

$$\left.\begin{array}{l} \eta = y\sqrt{\dfrac{a(n+1)}{2\nu}}x^{\frac{n-1}{2}}, \quad u = ax^n f'(\eta), \\ v = -\sqrt{\dfrac{a\nu(n+1)}{2}}x^{\frac{n-1}{2}}\left(f + \left(\dfrac{n-1}{n+1}\right)\eta f'\right) \\ \theta(\eta) = \dfrac{T - T_\infty}{T_w - T_\infty}, \quad \phi(\eta) = \dfrac{C - C_\infty}{C_w - C_\infty} \end{array}\right. \tag{9}$$

the fundamental equations of the boundary layer (3)–(6) are transformed to ordinary differential equations that are locally valid as follows:

$$f''' + ff'' - \left(\frac{2n}{n+1}\right)f'^2 = 0 \tag{10}$$

$$\frac{1}{Pr}\theta'' + f\theta' - \left(\frac{2r}{n+1}\right)f'\theta + Nb\theta'\phi' + Nt\theta'^2 = 0 \tag{11}$$

$$\phi'' + Lef\phi' + \frac{Nt}{Nb}\theta'' = 0 \tag{12}$$

In view of (9), the boundary conditions (7) turn into

$$\left.\begin{array}{l} f = F_w,\ f' = 1 + \zeta_p f'',\ \theta = 1,\ \phi = 1 \quad \text{at} \quad \eta = 0 \\ f' \to 0,\ \theta \to 0,\ \phi \to 0 \quad \text{as} \quad \eta \to \infty \end{array}\right\} \tag{13}$$