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ORIGINAL ARTICLE

A new modification of variational iteration method for solving reaction–diffusion system with fast reversible reaction

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Abstract This paper presents a new modification of He's variational iteration method using Adomian's polynomials (VIMAP) to solve reaction–diffusion system with fast reversible reaction. An auxiliary parameter is introduced into the VIMAP and optimally identified to adjust the convergence region of the approximate solution. The results reveal that the VIMAP is very accurate comparing with those obtained by the VIM but is not valid for large solution domain, while the new modification have a remarkable accuracy for large domains.

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1. Introduction

Nonlinear phenomena are of fundamental importance in various fields of science and engineering. The nonlinear models of real-life problems are still difficult to solve either numerically or theoretically. There has recently been much attention devoted to the search for better and more efficient solution methods for determining a solution, approximate or exact, analytical or numerical, to nonlinear models [1].

Many promising numeric–analytic methods have been proposed recently such as the variational iteration method (VIM) by He [2–5] and the Adomian's decomposition method (ADM) [6–9]. In recent years, many authors have successfully applied the VIM [10–13] to solve a wide variety of linear and nonlinear problems with approximations converging rapidly to accurate solutions. With the passage of time some modifications in He's variational iteration method (VIM) has been introduced by various authors [14–28].

The reaction diffusion equations (RDEs) have recently attracted considerable attention, partly due to their occurrence in many fields of science, in physics as well as in chemistry or biology, partly due to their interesting features and rich variety of properties of their solutions [29].

Recently, Eymard et al. [30] studied the numerical solution of the reaction–diffusion system with fast reversible chemical reaction of type $mA \rightleftharpoons nB$ by using the finite volume method.

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Also, Al-Sawoor and Al-Amr [31] applied the VIM and the ADM to solve this system and compared the obtained results.

The motivation of this paper is to present a new modification of the variational iteration method using Adomian's polynomials (VIMAP) by introducing an optimal auxiliary parameter into the VIMAP. The VIMAP and its modification are successfully applied to solve reaction–diffusion system which describes a reversible chemical reaction. Comparisons are made between the standard VIM, the VIMAP and the proposed method.

In this work, we consider a reversible chemical reaction between mobile species A and B, that takes place inside a bounded region $\Omega \subset \mathbb{R}$, we have the reaction–diffusion system of partial differential equations [30,32]:

$$\begin{aligned} u_t &= a\Delta u - \alpha k(r_A(u) - r_B(v)), \quad \text{in } \Omega \times (0,T), \\ v_t &= b\Delta v + \beta k(r_A(u) - r_B(v)), \quad \text{in } \Omega \times (0,T), \end{aligned} \tag{1}$$

where $T > 0$ and Ω is a bounded set of $\mathbb{R}$, with the boundary conditions

$$\nabla u \cdot n = \nabla v \cdot n = 0, \quad \text{on } \partial\Omega \times (0,T), \tag{2}$$

and the initial conditions

$$u(x,0) = u_0(x), \quad v(x,0) = v_0(x), \quad \text{in } \Omega. \tag{3}$$

For a reversible reaction $\alpha A \underset{k_2}{\overset{k_1}{\rightleftharpoons}} \beta B$, the rate functions are of the form $r_A(u) = k_1 u^\alpha$ and $r_B(u) = k_2\, v^\beta$, where k_1 and k_2 are rate constants, a and b are diffusion coefficients and k is the chemical kinetics factor (for further details see [33,34]).

2. Variational iteration method using Adomian's polynomials (VIMAP)

To illustrate the methodology of the VIMAP, we first consider the system of partial differential equations written in an operator form

$$\begin{aligned} L_t u + R_1(u,v) + N_1(u,v) &= g_1, \\ L_t v + R_2(u,v) + N_2(u,v) &= g_2, \end{aligned} \tag{4}$$

with initial data

$$\begin{aligned} u(x,0) &= f_1(x), \\ v(x,0) &= f_2(x), \end{aligned} \tag{5}$$

where L_t is considered, without loss of generality, a first order partial differential operator, R_1 and R_2 are linear operators, N_1 and N_2 are nonlinear operators, and g_1 and g_2 are inhomogeneous terms.

According to the VIM, we can construct a correctional functional as follows [11,13]:

$$\begin{aligned} u_{n+1}(x) &= u_n(x) + \int_0^x \lambda_1[L_t u_n(\tau) + R_1(\tilde{u}_n, \tilde{v}_n) + N_1(\tilde{u}_n, \tilde{v}_n) - g_1(\tau)]d\tau, \\ v_{n+1}(x) &= v_n(x) + \int_0^x \lambda_2[L_t v_n(\tau) + R_2(\tilde{u}_n, \tilde{v}_n) + N_2(\tilde{u}_n, \tilde{v}_n) - g_2(\tau)]d\tau, \end{aligned} \tag{6}$$

where λ_1 and λ_2 are general Lagrange multipliers, which can be identified optimally via the variational theory [5], the subscript n denotes the nth order approximation, $\tilde{u}_n$ and $\tilde{v}_n$ are considered as restricted variations, i.e., $\delta\tilde{u}_n = 0$ and $\delta\tilde{v}_n = 0$.

The ADM assumes a series that the unknown functions $u(x,t)$ and $v(x,t)$ can be expressed by an infinite series of the form [16,17]

$$\begin{aligned} u(x,t) &= \sum_{k=0}^{\infty} u_k(x,t), \\ v(x,t) &= \sum_{k=0}^{\infty} v_k(x,t). \end{aligned} \tag{7}$$

And the nonlinear operators $N_1(u,v)$ and $N_2(u,v)$ can be decomposed by the infinite series of the so-called Adomian polynomials

$$\begin{aligned} N_1(u,v) &= \sum_{k=0}^{\infty} A_k, \\ N_2(u,v) &= \sum_{k=0}^{\infty} B_k. \end{aligned} \tag{8}$$

The Adomian polynomials A_k and B_k are generated according to the following algorithms [16]

$$\begin{aligned} A_k &= \frac{1}{k!}\left[\frac{\partial^k}{\partial\lambda^k} N_1\left(\sum_{i=0}^{\infty}\lambda^i u_i, \sum_{i=0}^{\infty}\lambda^i v_i\right)\right]_{\lambda=0}, \\ B_k &= \frac{1}{k!}\left[\frac{\partial^k}{\partial\lambda^k} N_2\left(\sum_{i=0}^{\infty}\lambda^i u_i, \sum_{i=0}^{\infty}\lambda^i v_i\right)\right]_{\lambda=0}, \quad k > 0. \end{aligned} \tag{9}$$

Substituting Eqs. (7) and (8) into the variational iteration formula (6), we obtain

$$\begin{aligned} u_{n+1}(x) &= \int_0^x \lambda_1\left[\sum_{k=0}^{n} L_t u_k(\tau) + R_1\left(\sum_{k=0}^{n} u_k, \sum_{k=0}^{n} v_k\right) + \sum_{k=0}^{n} A_k - g_1(\tau)\right]d\tau, \\ v_{n+1}(x) &= \int_0^x \lambda_2\left[\sum_{k=0}^{n} L_t v_k(\tau) + R_2\left(\sum_{k=0}^{n} u_k, \sum_{k=0}^{n} v_k\right) + \sum_{k=0}^{n} B_k - g_2(\tau)\right]d\tau. \end{aligned} \tag{10}$$

The successive approximations $u_{n+1}(x,t)$, $v_{n+1}(x,t)$, $n \geqslant 0$, of the solutions $u(x,t)$ and $v(x,t)$ will be readily obtained by using selected functions f_1 and f_2. Consequently, the solutions are given by Eqs. (7).

3. Variational iteration method using Adomian's polynomials with an optimal auxiliary parameter (VIMAPOAP)

We assume that an unknown auxiliary parameter h can be inserted into the correction functional (10) of VIMAP, so that we obtain

$$\begin{aligned} u_{n+1}(x,h) &= h\int_0^x \lambda_1\left[\sum_{k=0}^{n} L_t u_k(\tau) + R_1\left(\sum_{k=0}^{n} \tilde{u}_k, \sum_{k=0}^{n} \tilde{v}_k\right) + \sum_{k=0}^{n} A_k - g_1(\tau)\right]d\tau, \\ v_{n+1}(x,h) &= h\int_0^x \lambda_2\left[\sum_{k=0}^{n} L_t v_k(\tau) + R_2\left(\sum_{k=0}^{n} \tilde{u}_k, \sum_{k=0}^{n} \tilde{v}_k\right) + \sum_{k=0}^{n} B_k - g_2(\tau)\right]d\tau. \end{aligned} \tag{11}$$

The auxiliary parameter h can be determined by means of the so-called h-curve and the error of norm 2 of the residual functions to ensures that the approximations $u_n(x,h)$, $v_n(x,h)$, $n \geqslant 1$, that contain the auxiliary parameter h, converge to the exact solutions. In fact, the proposed method gives a simple and a powerful mathematical tool for nonlinear problems and is cable to approximate the solution more accurately in a large solution domain.