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Results on n-tupled fixed points in complete asymptotically regular metric spaces

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Abstract The notion of n-tupled fixed point is introduced by Imdad, Soliman, Choudhury and Das, Jour. of Operators, Vol. 2013, Article ID 532867. In this manuscript, we prove some n-tupled fixed point theorems (for even n) for mappings having mixed monotone property in partially ordered complete asymptotically regular metric spaces. Our main theorem improves the corresponding results of Imdad, Sharma and Rao (M. Imdad, A. Sharma, K.P.R. Rao, Generalized n-tupled fixed point theorems for nonlinear contractions, preprint).

2010 MATHEMATICAL SUBJECT CLASSIFICATION: 47H10; 54H10; 54H25

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1. Introduction and preliminaries

The Banach contraction principle is the most natural and significant result of fixed point theory. It has become one of the most fundamental and powerful tools of nonlinear analysis because of its wide range of applications to nonlinear equations arising of in physical and biological processes ensuring the existence and uniqueness of solutions. It is widely considered as the source of metric fixed point theory. Also, its significance lies in its vast applicability in a number of branches of mathematics. Generalization of the above principle has been a heavily branch of mathematics. Existence of a fixed point for contraction type mappings in partially ordered metric space and applications have been considered by many authors. There already exists an extensive literature on this topic, but keeping in view the relevance of this paper, we merely refer to [1–13,17–31].

In [6], Bhaskar and Lakshmikantham introduced the notion of a coupled fixed point and proved some coupled fixed point theorems in partially ordered complete metric spaces under certain conditions. Afterwards Lakshmikantham and Ćirić [17] extended these results by defining the g-monotone property, which indeed generalize the corresponding fixed point theorems contained in [6]. Since then Berinde and Borcut [8] introduced the concept of tripled fixed point and proved some related theorems.

Recently Imdad et al. [14] introduced the concept of n-tupled coincidence as well as n-tupled fixed point (for even n) and utilize these two definitions to obtain n-tupled coincidence

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as well as n-tupled common fixed point theorems for nonlinear ϕ-contraction mappings. For more details see [9,11,23,25]. Very recently, Soliman et al. [31] proved some n-tupled coincident point theorems for nonlinear ϕ-contraction mappings in partially ordered complete asymptotically regular metric spaces.

The purpose of this paper is to present some n-tupled coincidence and fixed point results for nonlinear contraction mappings in partially ordered complete asymptotically regular metric spaces. Our results generalize and improve the results of Imdad et al. [15]. As usual, this section is devoted to preliminaries which include some basic definitions and results related to coupled fixed point in metric spaces.

From now, $(X, \preceq, d)$ is a partially ordered complete metric space. Further, the product space $X \times X$ has the following partial order:

$$(u, v) \preceq (x, y) \iff x \succeq u, y \preceq v \quad \forall (x, y), (u, v) \in X \times X.$$

We summarize in the following the basic notions and results established in [6,8].

Definition 1 [6]. Let $(X, \preceq)$ be a partially ordered set and $F: X \times X \to X$ be a mapping. Then F is said to have mixed monotone property if for any $x, y \in X$, $F(x, y)$ is monotonically nondecreasing in first argument and monotonically nonincreasing in second argument, that is, for

$$x_1, x_2 \in X, x_1 \preceq x_2 \Rightarrow F(x_1, y) \preceq F(x_2, y)$$
$$y_1, y_2 \in X, y_1 \preceq y_2 \Rightarrow F(x, y_1) \succeq F(x, y_2).$$

Definition 2 [6]. An element $(x, y) \in X \times X$ is called a coupled fixed point of the mapping $F: X \times X \to X$ if

$$F(x, y) = x \text{ and } F(y, x) = y.$$

The main theoretical results of Bhaskar and Lakshmikantham [6] are the following two coupled fixed point theorems.

Theorem 1. *Let $(X, \preceq)$ be a partially ordered set and suppose there is a metric d on X such that (X, d) is a complete metric space. Let $F: X \times X \to X$ be a continuous mapping having the mixed monotone property on X. Assume that there exists $k \in [0, 1)$ with*

$$d(F(x, y), F(u, v)) \leqslant \frac{k}{2}[d(x, u) + d(y, v)] \text{ for each } x \succeq u \text{ and } y \preceq v.$$

If there exist x_0, $y_0 \in X$ such that $x_0 \preceq F(x_0, y_0)$ and $y_0 \succeq F(y_0, x_0)$, then there exist x, $y \in X$ such that $F(x, y) = x$ and $F(y, x) = y$.

Theorem 2. *Let $(X, \preceq)$ be a partially ordered set and suppose there is a metric d on X such that (X, d) is a complete metric space. Assume that X has the following property:*

(i) *if nondecreasing sequence $\{x_n\} \to x$, then $x_n \preceq x$ for all n;*
(ii) *if nonincreasing sequence $\{y_n\} \to y$, then $y_n \succeq y$ for all n. Let $F: X \times X \to X$ be a mapping having the mixed monotone property on X. Assume that there exists $k \in [0, 1)$ with*

$$d(F(x, y), F(u, v)) \leqslant \frac{k}{2}[d(x, u) + d(y, v)] \text{ for each } x \succeq u \text{ and } y \preceq v.$$

If there exist x_0, $y_0 \in X$ such that $x_0 \preceq F(x_0, y_0)$ and $y_0 \succeq F(y_0, x_0)$, then there exist x, $y \in X$ such that $F(x, y) = x$ and $F(y, x) = y$.

Recently, Berinde and Borcut [8] introduced the following partial order on the product space $X \times X \times X$:

$$(u, v, w) \preceq (x, y, z) \iff x \succeq u, y \preceq v, z \succeq w$$
$$\forall (x, y, z), (u, v, w) \in X \times X \times X.$$

Definition 3 [8]. Let $(X, \preceq)$ be a partially ordered set and $F: X \times X \times X \to X$ be a mapping. Then F is said to have mixed monotone property if F is monotone nodecreasing in first and third argument and monotone noincreasing in second argument, that is, for any x, y, $z \in X$

$$x_1, x_2 \in X, x_1 \preceq x_2 \Rightarrow F(x_1, y, z) \preceq F(x_2, y, z)$$
$$y_1, y_2 \in X, y_1 \preceq y_2 \Rightarrow F(x, y_1, z) \succeq F(x, y_2, z)$$
$$z_1, z_2 \in X, z_1 \preceq z_2 \Rightarrow F(x, y, z_1) \preceq F(x, y, z_2).$$

Definition 4 [8]. An element $(x, y, z) \in X \times X \times X$ is called a tripled fixed point of the mapping $F: X \times X \times X \to X$ if $F(x, y, z) = x$, $F(y, x, y) = y$ and $F(z, y, x) = z$.

Inspired by the results on coupled fixed points, Karapinar [16] introduced the notion of quadrupled fixed point and proved some related fixed point theorems in partially ordered metric spaces.

Definition 5 [16]. An element $(x, y, z, w) \in X \times X \times X \times X$ is called a quadrupled fixed point of the mapping F: $X \times X \times X \times X \to X$ if $F(x, y, z, w) = x$, $F(y, z, w, x) = y$, $F(z, w, x, y) = z$ and $F(w, x, y, z) = w$.

The following concept of an n-fixed point was introduced by Gordji and Ramezani [12]. We suppose that the product space $X^n = X \times X \times \cdots \times X$($n$ times) is endowed with the following partial order, where n is the positive integer (odd or even): $(x^1, x^2, \ldots, x^n)$, $(y^1, y^2, \ldots, y^n) \in X^n$

$$(x^1, x^2, \ldots, x^n) \preceq (y^1, y^2, \ldots, y^n) \iff x^{2i-1} \preceq y^{2i-1} \ \forall i \in \left\{1, 2, \ldots, \left[\frac{n+1}{2}\right]\right\}$$
$$(x^1, x^2, \ldots, x^n) \preceq (y^1, y^2, \ldots, y^n) \iff x^{2i} \succeq y^{2i} \ \forall i \in \left\{1, 2, \ldots, \left[\frac{n}{2}\right]\right\}.$$

Definition 6 [12]. An element $(x^1, x^2, \ldots, x^n) \in X^n$ is called an n fixed point of the mapping $F: X^n \to X$ if

$$x^i = F(x^i, x^{i-1}, \ldots, x^2, x^1, x^2, \ldots, x^{n-i+1}) \ \forall i \in \{1, 2, \ldots, n\}.$$

Remark 1. The concept of n-tupled fixed point is given by Imdad et al. [14], which is quite different from the concept of Gordji and Ramezani [12]. A detailed version on n-tupled fixed point is given in next section.

Very recently, Soliman et al. [31] proved results on n-tupled coincidence point in complete asymptotically regular metric space and called this space as generalized complete metric space.

Some more definitions related to our script are as follows:

Definition 7. Let (X, d) be a metric space. Then a sequence $\{x_n\}$ in X is said to be Cauchy if $\lim_{n,m\to\infty} d(x_n, x_m) = 0$ for all $n, m \in \mathbb{N}$.

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