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Common fixed point theorems for hybrid pairs of maps in fuzzy metric spaces

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Abstract The purpose of this paper is to introduce the notion of common limit range property (CLR property) for two hybrid pairs of mappings in fuzzy metric spaces, and we prove common fixed point theorems using (CLR) property for these mappings with implicit relation. Our results extend some known results to multi-valued arena. Also, we prove common fixed point theorem in fuzzy metric spaces satisfying an integral type.

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1. Introduction and preliminaries

Fuzzy set [1] is an important concept in topology and analysis. It is a generalization of crisp set. The concept of fuzzy metric spaces has been studied by many authors in several ways. Kramosil and Michalek [2] introduced the concept of KM-fuzzy metric space as a generalization of probabilistic metric space given by Menger [3] and Schweizer and Sklar [4]. George and Veeramani [5] modified this concept to GV-fuzzy metric space and obtained a hausdorff topology for this kind of fuzzy metric spaces. Fuzzy set theory has applications in applied sciences such as mathematical programming, modeling theory, engineering sciences, image processing, control theory, and communication. Many authors have proved fixed and common fixed point theorems in metric and fuzzy metric spaces.

Mishra et al. [6] extended the notion of compatible maps under the name of asymptotically commuting maps in fuzzy metric spaces and prove common fixed point theorems using the continuity of one map and completeness of the involved maps. Singh and Jain [7] introduced the notion of weak and semicompatible maps in fuzzy metric spaces and showed that every pair of compatible maps is weakly compatible but the converse is not true in general. Pant [8] initiated the study of common fixed points of non-compatible maps in metric spaces. For a non-compatible maps, Aamri and El Moutawakil [9] introduced a new property named as (E.A) property, Pant [10] studied the common fixed points for non-compatible maps using (E.A) property in fuzzy metric spaces. Recently, Sintunavarat and Kumam [11] introduced the notion of common limit range property (or (CLR) property) for a pair of maps as a generalization of (E.A) property and prove

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common fixed point theorems in fuzzy metric spaces. The concept of (CLRg) property for hybrid maps is an extending of single maps. There are some similar results in deferent ways such as [12–14].

The aim of this paper is to extend some definitions and prove common fixed point theorems for hybrid maps in fuzzy spaces using the (CLRg) property. Our results are improvement over some relevant results contained in [15–19] besides some other ones.

Now we list some important definitions.

Definition 1.1 [20]. A binary operation $*:[0,1]\times[0,1]\rightarrow[0,1]$ is said to be continuous t-norm if

(I) $*$ is commutative and associative,
(II) $*$ is continuous,
(III) $a*1=a$ for all $a\in[0,1]$,
(IV) $a*b\leqslant c*d$ whenever $a\leqslant c$ and $b\leqslant d$ for all $a,b,c,d\in[0,1]$.

Example 1.2 [21]. As classical examples of continuous t-norms we mention the t-norms T_L, T_P, T_M defined through $T_L(a,b)=\max(a+b-1,0)$, $T_P(a,b)=ab$ and $T_M=\min(a,b)$.

Definition 1.3 [2]. The 3-tuple $(X,M,*)$ is said to be a KM-fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2\times[0,\infty)$ satisfying the following conditions (for all $x,y,z\in X$ and $t,s>0$):

(KM_1) $M(x,y,0)=0$,

(KM_2) $M(x,y,t)=1$ $\forall t>0$ iff $x=y$,

(KM_3) $M(x,y,t)=M(y,x,t)$,

(KM_4) $M(x,z,t+s)\geqslant M(x,y,t)*M(y,z,s)$,

(KM_5) $M(x,y,.):[0,\infty)\rightarrow[0,1]$ is left continuous.

Remark 1.4 [22]. The function $M(x,y,t)$ is often interpreted as the degree of nearness between x and y with respect to t.

Lemma 1.5 [23]. *For every $x,y\in X$, the mapping $M(x,y,\cdot)$ is nondecreasing on $(0,\infty)$.*

Definition 1.6 [5]. The 3-tuple $(X,M,*)$ is said to be a GV-fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm and M is a fuzzy set on $X^2\times(0,\infty)$ satisfying the following conditions (for all $x,y,z\in X$ and $t,s>0$):

$(GV_1) M(x,y,t)>0$,

$(GV_2) M(x,y,t)=1$ iff $x=y$,

$(GV_3) M(x,y,t)=M(y,x,t)$,

$(GV_4) M(x,z,t+s)\geqslant M(x,y,t)*M(y,z,s)$,

$(GV_5) M(x,y,.):(0,\infty)\rightarrow[0,1]$ is continuous.

Example 1.7 [5]. Let (X,d) be a metric space wherein $a*b=ab$ for all $a,b\in[0,1]$. Then, one can define a fuzzy metric $M_d(x,y,t)$ by

$$M_d(x,y,t)=\frac{t}{t+d(x,y)},$$

for each $x,y\in X$ and $t>0$.

Definition 1.7 [24]. Let $CB(X)$ be the set of all nonempty closed bounded subsets of a fuzzy metric space $(X,M,*)$. Then for every $A,B,C\in CB(X)$ and $t>0$,

$$M(A,B,t)=\min\left\{\min_{a\in A}M(a,B,t),\ \min_{b\in B}M(A,b,t)\right\},$$

where $M(C,y,t)=\max\{M(z,y,t):z\in C\}$.

Remark 1.8 [16]. Obviously $M(A,B,t)\leqslant M(a,B,t)$ whenever $a\in A$ and $M(A,B,t)=1$ iff $A=B$. Obviously, $1=M(A,B,t)\leqslant M(a,B,t)$ for all $a\in A$.

Definition 1.24 [25]. Let $(X,M,*)$ be a fuzzy metric space. We denote by $CP(X)$ the set of nonempty compact subsets of X. We define a function H_M on $CP(X)\times CP(X)\times(0,\infty)$ by

$$H_M(A,B,t)=\min\left\{\inf_{a\in A}M(a,B,t),\ \inf_{b\in B}M(A,b,t)\right\},$$

for all $A,B\in CP(X)$ and $t>0$, also $(H_M,*)$ is a fuzzy metric on $CP(X)$.

Definition 1.9 [5]. A sequence $\{x_n\}$ in a fuzzy metric space $(X,M,*)$ is said to be convergent to some $x\in X$ if for all $t>0$, $\lim_{n\rightarrow\infty}M(x_n,x,t)=1$.

Definition 1.10 [26]. Let $CL(X)$ be the set of all nonempty closed subsets of a metric space (X,d) and $F:Y\subseteq X\rightarrow CL(X)$. Then the map $f:Y\rightarrow X$ is said to be F-weakly commuting at $x\in X$ if $ffx\in Ffx$ provided that $fx\in Y$ for all $x\in Y$.

Definition 1.12 [27]. Let $(X,M,*)$ be a fuzzy metric space. A map $f:Y\subseteq X\rightarrow X$ is said to be coincidentally idempotent w.r.t. a mapping $F:Y\rightarrow CL(X)$ if f is idempotent at the coincidence points of (f,F), i.e., $ffx=fx$ for all $x\in Y$ with $fx\in Fx$ provided that $fx\in Y$.

Definition 1.13 [11]. Let (X,d) be metric space. Two mappings $f,g:X\rightarrow X$ are said to be satisfy the (CLR_g) property if there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\rightarrow\infty}fx_n=\lim_{n\rightarrow\infty}gx_n=gx,$$

for some $x\in X$.

Motivated from Definition 1.13, we can have

Definition 1.14. Let (X,d) be metric space. Two mappings $f:X\rightarrow X$, $F:X\rightarrow CL(X)$ are said to be satisfy the property (CLR_f) if there exist sequence $\{x_n\}$ in X such that