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Regular open sets in fuzzifying topology redefined

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Abstract In 2000 [1], Zahran introduced the concept of regular open sets in fuzzifying topology. In 2004 [2], Sayed and Zahran, gave an example to illustrate that the statements:

(1) $\models A \in R_\tau \to A \in \tau$ (Lemma 2.2 [1]); and
(2) $\models (A \in R_\tau \wedge B \in R_\tau) \to A \cap B \in R_\tau$ (Theorem 2.4 [1]),

are incorrect. In the present paper we redefine this concept to make these statements correct. Furthermore, by making use of our definition of regular open sets, the concepts of almost continuity and δ-continuity are introduced and studied in fuzzifying topology.

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1. Introduction

In classical and fuzzy topology, Almost continuity, δ-continuity [3–5] have been defined and their properties have been obtained.

In 1991 [6], Ying used the semantics of fuzzy logic to propose a topology whose logical fundament is fuzzy. Proceeding in this direction many papers have been written [1,7–9]. The concept of regular open set in fuzzifying topology was given in 2000 [1] by Zahran. In 2004 [2], Sayed and Zahran illustrate by a counterexample that the statements:

(1) $\tau(A) \geqslant R_\tau(A)$ (Lemma 2.2 [1]); and
(2) $(R_\tau(A) \wedge R_\tau(B)) \leqslant R_\tau(A \cap B)$ (Theorem 2.4 [1]),

are incorrect. In the present paper we redefine the concept of regular open sets in fuzzifying topology to make these statements correct. Furthermore by making use of this concept we introduce and study the almost continuity and δ-continuity in fuzzifying topology.

For the definition of a fuzzifying topology and some of its basic concepts used in this paper we refer to [6,8,9]. For the definitions of the family of semi-open sets and the family of semi-closed sets in fuzzifying topology we refer to [7].

However we recall here some of the basic concepts used in this paper.

Definition 1.1. Let (X, τ) be a fuzzifying topological space. Then

(1) The family of all closed sets in X is denoted by F_τ or F if there is no confusion and defined as: $F_\tau(A) = \tau(X - A)\ \forall\ A \in 2^X$, where $X - A$ is the complement of A.
(2) The neighborhood system of x at a subset A of X is denoted by $\phi_{(\tau,x)}(A)$, and defined as:

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$$\phi_{(\tau,x)}(A) = \bigvee_{x \in B \subseteq A} \tau(B) \quad \forall\ A \in 2^X.$$

(3) The closure (resp. interior) of A is denoted by $cl_\tau(A)$, (resp. $int_\tau(A)$,) and defined as:

$$cl_\tau(A)(x) = 1 - \phi_{(\tau,x)}(X - A)(resp.\ int_\tau(A)(x) = \phi_{(\tau,x)}(A))\ \forall\ A \in 2^X, \quad \forall\ x \in X.$$

(4) Let $f \in I^X$, where $I = [0, 1]$. Then
(a) The closure of f is denoted by $\widetilde{cl}_\tau(f)$, and defined as:

$$\widetilde{cl}_\tau(f)(x) = \bigvee_{\alpha \in [0,1]} (f(x) \wedge cl_\tau(f_\alpha))(x) \quad \forall\ x \in X;\ and$$

(b) The interior of f is denoted by $\widetilde{int}_\tau(f)$, and defined as:

$$\widetilde{int}_\tau(f) = 1 - \widetilde{cl}_\tau(1 - f).$$

(5) The family of semi-open sets is denoted by $S\tau$, and defined as:

$$S\tau(A) = \bigwedge_{x \in A} \widetilde{cl}_\tau(int_\tau(A))(x) \quad \forall\ A \in 2^X.$$

(6) The family of semi-closed sets is denoted by SF, and defined as:

$$SF(A) = S\tau(X - A) \quad \forall\ A \in 2^X.$$

(7) The degree of the convergence of a net S in X to $x \in X$ is denoted by $S \rhd_\tau x$, and defined as:

$$S \rhd_\tau x = \bigwedge_{S \not\subseteq A} (1 - \phi_{(\tau,x)}(A))$$

$\forall\ S \in N(X), \forall\ x \in X$, where $S \subseteq A$ means S almost in A and $N(X)$ denoted the set of all nets in X.

Definition 1.2. Let $f, g \in I^X$. The fuzzy inclusion of f in g is denoted by $[[f, g[[$, and defined as:

$$[[f, g[[= \bigwedge_{x \in X} (f(x) \to g(x)).$$

Note, that "$\to$" is defined by: $\alpha \to \beta = \min(1, 1 - \alpha + \beta)\ \alpha,\ \beta \in I$.

2. Regular open sets and δ-open sets

Definition 2.1. Let (X, τ) be a fuzzifying topological space. Then

(1) The family of all regular open sets is denoted by $R\tau \in I^{(2^X)}$ and defined as follows:

$$R\tau(A) = \tau(A) \wedge SF(A).$$

(2) The family of all regular closed sets is denoted by $RF \in I^{(2^X)}$ and defined as follows:

$$RF(A) = R\tau(X - A) \quad \forall\ A \in 2^X.$$

Theorem 2.1. *Let (X, τ) be a fuzzifying topological space. Then*

(1) (a) $R\tau(X) = 1, R\tau(\phi) = 1$;
(b) $R\tau(A \cap B) \geqslant R\tau(A) \wedge R\tau(B)$;
(c) $\tau(A) \geqslant R\tau(A), SF(A) \geqslant R\tau(A)$;

(2) (a) $RF(X) = 1, RF(\phi) = 1$;
(b) $RF(A \cup B) \geqslant RF(A) \wedge RF(B)$;
(c) $F(A) \geqslant RF(A), S\tau(A) \geqslant RF(A)$;
(d) $RF(A) = F(A) \wedge S\tau(A)$.

Proof. We just prove (1) (b). From Theorem 3.2 (1) (b) [7], we have

$$R\tau(A \cap B) = \tau(A \cap B) \wedge SF(A \cap B) \geqslant \tau(A) \wedge \tau(B) \wedge SF(A) \wedge SF(B) = R\tau(A) \wedge R\tau(B).$$

The other statements are clear. □

In 2004 [2], Sayed and Zahran illustrate by the following example that the statements:

(1) $\tau(A) \geqslant R\tau(A)$ (Lemma 2.2 [1]); and
(2) $(R\tau(A) \wedge R\tau(B)) \leqslant R\tau(A \cap B)$ (Theorem 2.4 [1]), are incorrect.

Example 2.1. Let $X = \{a, b, c\}$ and τ be a fuzzifying topology on X defined as $\tau(X) = \tau(\emptyset) = \tau(\{a\}) = \tau(\{a, c\}) = 1$, $\tau(\{b\}) = \tau(\{a, b\}) = 0$ and $\tau(\{c\}) = \tau(\{b, c\}) = \frac{1}{8}$.

Sayed and Zahran have obtained the regular openness degree of every $A \in 2^X$ according to the definition of regular open in form $R\tau(A) = (A \equiv int_\tau(cl_\tau(A)))$ as follows:

$R\tau(X) = R\tau(\emptyset) = 1$, $R\tau(\{a\}) = R\tau(\{c\}) = R\tau(\{a, b\}) = R\tau(\{b, c\}) = \frac{1}{8}$ and $R\tau(\{b\}) = R\tau(\{a, c\}) = 0$. Therefore, as we see $R\tau(\{a, b\}) > \tau(\{a, b\})$ and $R\tau(\{a, b\} \cap \{b, c\}) < R\tau(\{a, b\}) \cap R\tau(\{b, c\})$.

Now, we obtain the regular openness degree of every $A \in 2^X$ according to the definition of regular open in form $R\tau(A) = \tau(A) \wedge SF(A)$ as follows:

Example 2.2. Let $X = \{a, b, c\}$ and τ be a fuzzifying topology on X that defined in Example 2.1. So, $SF(X) = SF(\emptyset) = SF(\{b\}) = 1$, $SF(\{a\}) = SF(\{a, b\}) = SF(\{a, c\}) = 0$ and $SF(\{c\}) = SF(\{b, c\}) = \frac{7}{8}$. Therefore, as we see $R\tau(X) = R\tau(\emptyset) = 1$, $R\tau(\{a\}) = R\tau(\{b\}) = R\tau(\{a, b\}) = R\tau(\{a, c\}) = 0$ and $R\tau(\{c\}) = R\tau(\{b, c\}) = \frac{1}{8}$. Thus $R\tau(A) \leqslant \tau(A)$ for every $A \in 2^X$ and $R\tau(A) \cap R\tau(B) \leqslant R\tau(A \cap B)$ for every $A, B \in 2^X$.

Definition 2.2. Let (X, τ) be a fuzzifying topological space and let $x \in X$. The δ-neighborhood system of x is denoted by $\delta\phi_{(\tau,x)} \in I^{(2^X)}$ and defined as follows:

$$\delta\phi_{(\tau,x)}(A) = \bigvee_{x \in B \subseteq A} R\tau(B) \quad \forall\ A \in 2^X.$$