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ORIGINAL ARTICLE

Unsteady mixed convection heat transfer along a vertical stretching surface with variable viscosity and viscous dissipation

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Abstract The effect of variable viscosity on laminar mixed convection flow and heat transfer along a semi-infinite unsteady stretching sheet taking into account the effect of viscous dissipation is studied. The flow of the fluid and subsequent heat transfer from the stretching surface is investigated with the aid of suitable transformation variables. Solutions for the velocity and temperature fields are obtained for some representative values of the unsteadiness parameter, variable viscosity parameter, mixed convection parameter and Eckert number. Typical velocity and temperature profiles, the local skin friction coefficient and the local heat transfer rate are presented at selected controlling parameters.

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1. Introduction

Boundary layer flow on a moving continuous surface is an important type of flow occurring in a number of engineering processes. Aerodynamic extrusion of plastic sheets, cooling of metallic sheets in a cooling bath, which would be in the form of an electrolyte, crystal growing, the boundary layer along a liquid film in condensation process and polymer sheet extruded continuously from a die are the practical applications of moving surfaces and also the materials manufactured by extrusion processes and heat treated materials traveling between a feed roll and wind up roll or on a conveyer belt possesses the characteristics of a moving continuous surfaces. After a pioneering work of Sakiadis [1,2] the study of flow and heat transfer characteristics past continuous stretching surfaces has drawn considerable attention, and a good amount of the literature has been generated on this problem (see for instance [3–18]). Although various aspects of this class of boundary layer problems have been tackled, the effect of buoyancy force was ignored in the above-cited reports

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[1–18]. In actual practice, the flow over a continuous material moving through a quiescent fluid is induced by the movement of the solid material and by thermal buoyancy. Therefore, these two mechanisms, surface motion and buoyancy force will determine the momentum and thermal transport processes. The thermal buoyancy force arising due to the heating of a continuously moving surface, under some circumstances, may alter significantly the flow and thermal fields and thereby the heat transfer behavior in the manufacturing process. Lin et al. [19] studied the problem of mixed convection from an isothermal horizontal plate moving in parallel or reversely to a free stream. Ali and Al-Yousef [20–21] investigated the problem of laminar mixed convection adjacent to a moving vertical surface with suction or injection. Karwe and Jaluria [22] showed that the thermal buoyancy effects are more prominent when the plate moves vertically, i.e., aligned with the gravity, than when it is horizontal. Also, an analysis of mixed convection heat transfer from a vertical continuously stretching sheet has been presented by Chen [23]. Ali [24] studied the buoyancy effect on the boundary layer induced by continuous surface stretched with rapidly decreasing velocities. Abo-Eldahab and Abd El-Aziz [25] presented the problem of steady, laminar, hydromagnetic heat transfer by mixed convection over an inclined stretching surface in the presence of space- and temperature-dependent heat generation or absorption effects. Abd El-Aziz [26] investigated the problem of thermal radiation effects on magnetohydrodynamic mixed convection flow of a micropolar fluid past a continuously moving semi-infinite plate for high temperature differences. Abd El-Aziz and Salem [27] studied the effects of coupled heat and mass transfer by natural convection and chemical reaction on the flow of an incompressible, viscous, and electrically conducting fluid past a vertical and linearly stretched permeable sheet with variable surface temperature and mass flux. Salem and Abd El-Aziz [28] investigated the effect of Hall currents and chemical reaction on hydromagnetic flow of a stretching vertical surface with internal heat generation or absorption. In all the above mentioned studies the viscosity of the ambient fluid was assumed to be constant. However, it is known that the fluid viscosity changes with temperature [29], for example the absolute viscosity of water decreases by 240% when the temperature increases from 10 °C to 50 °C. So in order to predict accurately the flow behavior, it is necessary to take into account this variation in viscosity since recent results on the flow due to stretching surface with and without buoyancy force have shown that when this effect is included, the flow characteristics may be substantially changed compared to the constant viscosity case. Pop et al. [30] studied the effect of variable viscosity on flow and heat transfer to a continuous moving flat plate using the similarity solution with no buoyancy force. Mukhopadhyay et al. [31] studied the boundary layer flow over a heated stretching sheet with variable viscosity in the presence of magnetic field. Ali [32] studied the effect of temperature-dependent viscosity on mixed convection heat transfer along a vertical moving surface taking into account the effect of buoyancy force. Abd El-Aziz [33] studied the problem of temperature-dependent viscosity and thermal conductivity effects on combined heat and mass transfer in MHD three-dimensional flow over a stretching surface with Ohmic heating. If the fluid is viscous, considerable heat can be produced even though at relatively low speeds, e.g. in the extrusion of a plastic, and hence the heat transfer results may alter appreciably due to viscous dissipation. Partha et al. [34] presented a similarity solution for mixed convection flow and heat transfer from an exponentially stretching surface by considering viscous dissipation effect in the medium. All of the above investigations have been restricted to steady-state conditions. However, in certain practical problems, the motion of the stretched surface may start impulsively from rest. In these problems the transient or unsteady aspects become of interest. Elbashbeshy and Bazid [35] presented an exact similarity solution for momentum and heat transfer in an unsteady flow whose motion is caused solely by the linear stretching of a horizontal stretching surface. Ali and Magyari [36] presented the problem of unsteady fluid and heat flow induced by a submerged stretching surface while its steady motion is slowed down gradually. Recently, Abd El-Aziz [37–39] extended the problem of Elbashbeshy and Bazid [35] to include various aspects such as thermal radiation, Hall currents and time-dependent chemical reaction. However, the combined effect of buoyancy force, viscous dissipation and variable viscosity on the flow and heat transfer which is important in view point of desired properties of the outcome is not considered in the earlier papers [35–39]. Motivated by all the works mentioned above, it is of interest to extend the work [35] by including such effects on the flow and heat transfer from an unsteady stretching surface. It will be demonstrated that the system of time-dependent governing equations can be reduced to a five-parameter problem by introducing a suitable transformation variables. Accurate numerical solutions are generated by employing shooting method. A comprehensive parametric study is conducted and a representative set of graphical results for the velocity and temperature profiles as well as the skin friction and wall heat transfer coefficients are reported and discussed. The analysis showed that the unsteadiness parameter, buoyancy force, variable viscosity property and viscous dissipation have significant influence on the flow and thermal fields as well as the non-dimensional local skin friction and heat transfer coefficients.

2. Analysis

Consider the mixed convection, boundary layer flow of a viscous, incompressible fluid along an unsteady stretching sheet, which issues vertically in the upward direction from a slot with velocity

$$u_w = bx/(1 - \alpha t), \tag{1}$$

where b and α are positive constants with dimensions $(\text{time})^{-1}$. Here, b is the initial stretching rate, whereas the effective stretching rate $b/(1 - \alpha t)$ is increasing with time. In the context of polymer extrusion the material properties and in particular the elasticity of the extruded sheet may vary with time even though the sheet is being pulled by a constant force. A schematic representation of the physical model and coordinates system is depicted in Fig. 1. The positive x coordinate is measured along the stretching sheet with the slot as the origin and the positive y coordinate is measured normal to the sheet in the outward direction toward the fluid. The surface temperature T_w of the stretching sheet varies with the distance x from the slot and time t as [40]