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ORIGINAL ARTICLE

A mathematical model on Acquired Immunodeficiency Syndrome

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Abstract A mathematical model SEIA (susceptible-exposed-infectious-AIDS infected) with vertical transmission of AIDS epidemic is formulated. AIDS is one of the largest health problems, the world is currently facing. Even with anti-retroviral therapies (ART), many resource-constrained countries are unable to meet the treatment needs of their infected populations. We consider a function of number of AIDS cases in a community with an inverse relation. A stated theorem with proof and an example to illustrate it, is given to find the equilibrium points of the model. The disease-free equilibrium of the model is investigated by finding next generation matrix and basic reproduction number $\Re_0$ of the model. The disease-free equilibrium of the AIDS model system is locally asymptotically stable if $\Re_0 \leqslant 1$ and unstable if $\Re_0 > 1$. Finally, numerical simulations are presented to illustrate the results.

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1. Introduction

AIDS was first identified as a distinct new disease in 1981. In 1983 HIV was identified at the causative agent for AIDS. The mean time from HIV infection to AIDS is approximately 10 years. There is no effective medicine to cure it and the infected individuals do not recover: that is, they continue to be infectious throughout their lives. HIV infection is a complex mix of diverse epidemics within and between countries and regions of the world, and is undoubtedly the defining public health crisis of our time. The three known modes of transmission of HIV are

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sexual contact, direct contact with HIV-infected blood or fluids, and perinatal transmission from an infected mother to child [1,2].

The complexity of the complete formalization of mathematical models is probably responsible for the discomfort with which biologists treat mathematical research literature on medicine and biology. They also appear to have reservations about the convergence between epidemiological and their mathematical models. However, there are instances of success in constructing models capable of predicting the outcome of infections. Such instances inspire our confidence in proposing realistic models. Hence present modeling attempts should be aimed at determining HIV spread mechanisms through testing the sensitivity of the assumptions [3].

Anderson and May [3] present more models of infections including HIV with illustrations. Models of HIV spread specific to the type of transmission have also appeared in the literature. Mathematical models have also been developed recently for other sexually transmitted diseases (STDs). Garnett [4] has presented a simple and useful discussion on various mathematical models for STDs. Mukherji [5] represents one of the earliest Indian attempts at modelling data on AIDS. This model used annualized south Asian regional data and extrapolated to AIDS in future. Basu [6] attempt to model the spread of AIDS in a comprehensive manner with limited data. The incubation period of AIDS in India, estimated through deconvoluting HIV epidemic density and reported AIDS cases, is between 8 and 12 years. Quantitative information on commercial sex activity and female commercial sex workers (FSWs) number in India are available in various sources [7].

In the last decades, many mathematical models have been developed to describe the immunological response to infection with human immunodeficiency virus (HIV), for example, [8–12] and so on. These models have been used to explain different phenomena. For more references and some detailed mathematical analysis on such models, we refer to the survey papers by Kirschner [13] and Perelson and Nelson [14]. Waziri [15] developed a mathematical model of AIDS dynamics with treatment and vertical transmission and recently Miron & Smith [16] use mathematical modeling to describe the interaction between T cells, HIV-1 and protease inhibitors.

In this paper, we develop a mathematical model SEIA (susceptible- exposed-infectious-AIDS infected) of AIDS epidemic with vertical transmission. A method for finding the equilibrium point given the transmission term η(A) was provided through a proof of a theorem. An example to illustrate use of the theorem is also given. The disease-free equilibrium of the model is investigated by finding next generation matrix and basic reproduction number $\Re_0$ of the model.

The paper is organized as follows: Introduction is given in Section 1, the basic assumptions and parameters of the model is discussed in Section 2, the epidemic model is developed in section 3, Section 4 establishes the stability of the system developed, numerical simulations is given in Section 5, and finally conclusion in Section 6.

2. Model parameter and assumptions

We formulate an AIDS model with vertical transmission by considering the adult population in one group. At time t, there are $S(t)$ adult susceptibles, $E(t)$ latent or exposed phase, during which the individual is said to be infected but not infectious, $I(t)$ infectives who are the infected and infectious individuals that have not yet developed AIDS symptoms and $A(t)$ AIDS cases who are infected and with AIDS symptoms. Susceptibles have sexual contacts at a rate with a probability of transmission at one sexual encounter denoted by β. A proportion of these sexual contacts are with infectives. Assume that this proportion is equal to the prevalence of infectives in the population. The model upholds the common assumption of assuming no sexual contacts with AIDS cases though the role of sexual contacts with persons with AIDS symptoms may become important with advances in medical interventions. Sexual contacts within susceptibles do not result in any transmission and thus do not feature in the model. Also, sexual contacts within infectives which gives rise to issues about the role of reinfection are ignored. Denote the probability of transmission β and the contact rate c at the onset of the epidemic as β_0 and c_0, respectively. Assume that, at future points in time differs from $\beta_0 c_0$ at the beginning of the epidemic and that there is an inverse relationship between βc and the number of AIDS cases, i.e. βc decreases with an increasing number of AIDS cases and increases with decreasing numbers of AIDS cases. The decrease in with increases in number of AIDS cases is attributed to awareness and behaviour change. The increase in βc with decreasing numbers of AIDS cases is attributed to complacency. Our model allows for a generalized form of βc, which as per the above assumptions is a function of the number of AIDS cases denoted by $\eta(A)$. The objective is to investigate the implication of the dependence of the risk of transmission of HIV on the number of persons with AIDS symptoms in a community [17,18].

3. Model equations

In this section the SEIA model including vertical transmission is explained along with an exploration of the differential equations describing the flow from one compartment to another. The flow of this model is depicted in Fig. 1, and the system of equations as per our assumption is as follows:

$$\frac{dS}{dt} = \Lambda(t) - \eta(A)SI/N - \mu S \tag{I}$$

$$\frac{dE}{dt} = \eta(A)SI/N - \mu E - \delta E \tag{II}$$

$$\frac{dI}{dt} = \delta E + \varepsilon I - (\mu + \gamma + \xi)I \tag{III}$$

$$\frac{dA}{dt} = \xi I - (\mu + \sigma)A \tag{IV}$$

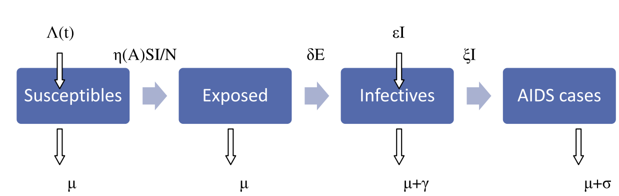

Figure 1 Schematic diagram for the flow of AIDS in the population.