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ORIGINAL ARTICLE

Circuit realization, chaos synchronization and estimation of parameters of a hyperchaotic system with unknown parameters

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Abstract In this article, the adaptive chaos synchronization technique is implemented by an electronic circuit and applied to the hyperchaotic system proposed by Chen et al. We consider the more realistic and practical case where all the parameters of the master system are unknowns. We propose and implement an electronic circuit that performs the estimation of the unknown parameters and the updating of the parameters of the slave system automatically, and hence it achieves the synchronization. To the best of our knowledge, this is the first attempt to implement a circuit that estimates the values of the unknown parameters of chaotic system and achieves synchronization. The proposed circuit has a variety of suitable real applications related to chaos encryption and cryptography. The outputs of the implemented circuits and numerical simulation results are shown to view the performance of the synchronized system and the proposed circuit.

MATHEMATICS SUBJECT CLASSIFICATION: 34H10; 34C28; 37M05

1. Introduction

The field of studying dynamical systems has attracted much attention and research activity in recent years. Dynamical systems play an important role in studying various phenomena that undergo spatial and temporal evolution. These phenomena arise from different disciplines including mechanical engineering, electrical engineering, physics, chemistry, biology, and economy [1,2].

Chaos is a very complex nonlinear behavior exists in some dynamical systems and it has some interesting properties such as complicated topological structure, high sensitivity to changes in initial conditions and system parameters. Chaos is characterized by having one positive Lyapunov exponent [3]. The modern applications of chaos and dynamical systems include chaos control, chaos synchronization, electronic circuits, secure communications, image encryption, cryptography, and neuroscience research [4–13].

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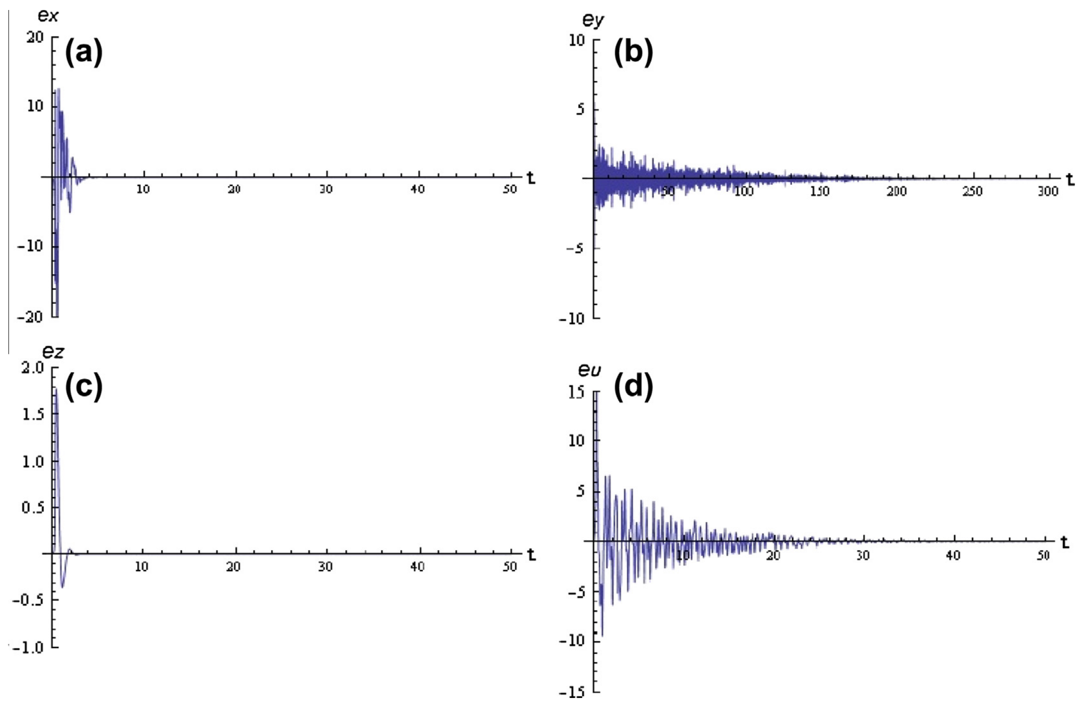

Figure 1 The error dynamics of the state variables (a) x, (b) y, (c) z and (d) u.

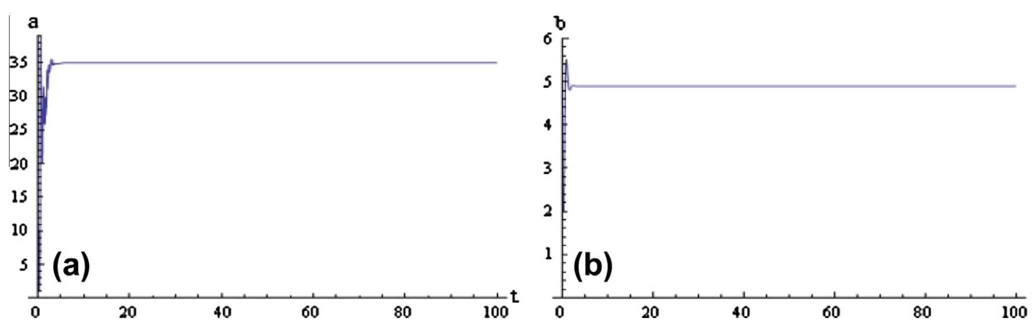

Figure 2 The estimation of the values of parameter a and b of the hyperchaotic system.

A hyperchaotic system is a system that is characterized by having more than one positive Lyapunov exponents for its chaotic attractor. Hyperchaos was firstly introduced by Rossler [14] then several hyperchaotic systems are introduced and studied extensively (see for example [3,15–18], and references therein). The hyperchaotic systems have higher unpredictability and more randomness than simple chaotic systems. So, hyperchaos is preferred in many applications including secure communications, chaos based image encryption, and cryptography.

In [15], the following fourth order hyperchaotic system was introduced

$$
\begin{aligned}
\dot{x} &= a(y - x) + hyz, \\
\dot{y} &= cx - lxz + y + u, \\
\dot{z} &= xy - bz, \\
\dot{u} &= -ky,
\end{aligned}
\tag{1}
$$

where x, y, z, and u are the state variables of the system and the constants a, b, c, l, h, and k are the parameters of the system to be tuned. This system has only one equilibrium point that is $(0, 0, 0, 0)$. As this hyperchaotic system has larger positive Lyapunov exponents than the already known hyperchaotic systems [15], and hence higher complexity and unpredictability, the presented circuit can be considered as a possible ideal choice for chaos generation in telecommunication systems. Also, the hyperchaotic behavior exists within a large range of the six parameters of the hyperchaotic system which gives the system the advantage of possessing a large domain of secret keys in real applications of chaotic based image encryption.

Chaos control and synchronization of system (1) are studied in [16,17]. In [16], hybrid projective synchronization is applied to the hyperchaotic system (1) whereas in [17], adaptive chaos synchronization is presented between two identical drive and response system with uncertain parameters. In this work, we investigate chaos synchronization in the practical and more realistic case that occurs when all the parameters of the drive system are unknown and the values of the parameters of the response system are not determined yet. During the process of synchronization, the values of the parameters of the drive