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ORIGINAL ARTICLE

Traveling wave solutions of the nonlinear Drinfel'd–Sokolov–Wilson equation and modified Benjamin–Bona–Mahony equations

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Abstract In this paper, the modified simple equation (MSE) method is implemented to find the exact solutions for the nonlinear Drinfel'd–Sokolov–Wilson (DSW) equation and the modified Benjamin–Bona–Mahony (mBBM) equations. The efficiency of this method for constructing these exact solutions has been demonstrated. It is shown that the MSE method is direct, effective and can be used for many other nonlinear evolution equations (NLEEs) in mathematical physics. Moreover, this technique reduces the large volume of calculations.

MATHEMATICS SUBJECT CLASSIFICATION: 35C07, 35C08, 35P99

1. Introduction

It is well known that most of the phenomena that arise in mathematical physics and engineering fields can be described by NLEEs. NLEEs have become a useful tool for describing natural phenomena of science and engineering models. NLEEs are frequently used to describe many problems of protein chemistry, chemically reactive materials, in ecology most population models, in physics the heat flow and the wave propagation phenomena, quantum mechanics, fluid mechanics, plasma physics, propagation of shallow water waves, optical fibers, biology, solid state physics, chemical kinematics, geochemistry, meteorology, electricity etc. By the aid of exact solutions, when they exist, the phenomena modeled by these NLEEs can be better understood. Therefore, the studies of the traveling wave solutions for NLEEs play an important role in the study of nonlinear physical phenomena. Considerable efforts have been made by many mathematicians and physical scientists to obtain exact solutions of such NLEEs and a number of powerful and efficient methods have been developed by those authors such as the Hirota's bilinear transformation method [1,2], the tanh-function method [3,4], the extended tanh-method [5,6], the Exp-function method [7–14], the Adomian decomposition method [15], the F-expansion method [16], the

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auxiliary equation method [17], the Jacobi elliptic function method [18], Modified Exp-function method [19], the (G'/G)-expansion method [20–29], Weierstrass elliptic function method [30], the homotopy perturbation method [31–35], the homogeneous balance method [36,37], the modified simple equation method [38–40], He's polynomial [41], the asymptotic methods [42], the variational iteration method [43,44], the casoration formulation [45], the Frobenius integrable decomposition method [46] and so on.

The objective of this article is to apply the MSE method to construct the exact solutions for nonlinear evolution equations in mathematical physics via nonlinear DSW equation and the mBBM equation.

The article is prepared as follows: In Section 2, the MSE method is discussed; In Section 3, we exert this method to the nonlinear evolution equations pointed out above; in Section 4, physical explanation and in Section 5 conclusions are given.

2. The MSE method

In this section we describe the MSE method for finding traveling wave solutions of nonlinear evolution equations. Suppose that a nonlinear equation, say in two independent variables x and t is given by

$$\mathscr{R}(u, u_t, u_x, u_{tt}, u_{xx}, u_{xt}, \ldots) = 0, \tag{2.1}$$

where $u(\xi) = u(x, t)$ is an unknown function, $\mathscr{R}$ is a polynomial of $u(x, t)$ and its partial derivatives in which the highest order derivatives and nonlinear terms are involved. In the following, we give the main steps of this method [38–40]:

Step 1. Combining the independent variables x and t into one variable $\xi = x \pm \omega t$, we suppose that

$$u(\xi) = u(x,t), \quad \xi = x \pm \omega t. \tag{2.2}$$

The traveling wave transformation Eq. (2.2) permits us to reduce Eq. (2.1) to the following ODE:

$$\mathscr{R}(u, u', u'', \ldots) = 0, \tag{2.3}$$

where $\mathscr{R}$ is a polynomial in $u(\xi)$ and its derivatives, while $u\prime(\xi) = \frac{du}{d\xi}$, $u\prime\prime(\xi) = \frac{d^2u}{d\xi^2}$, and so on.

Step 2. We suppose that Eq. (2.3) has the formal solution

$$u(\xi) = \alpha_0 + \sum_{k=1}^{n} \alpha_k \left(\frac{\phi'(\xi)}{\phi(\xi)}\right)^k, \tag{2.4}$$

where α_k are arbitrary constants to be determined, such that $\alpha_n \neq 0$ and $\phi(\xi)$ is an unknown function to be determined later.

Step 3. We determine the positive integer n in Eq. (2.4) by considering the homogeneous balance between the highest order derivatives and the nonlinear terms in Eq. (2.3).

Step 4. We substitute Eq. (2.4) into Eq. (2.3) and then we account the function $\phi(\xi)$. As a result of this substitution, we get a polynomial of $(\phi'(\xi)/\phi(\xi))$ and its derivatives. In this polynomial, we equate the coefficients of same power of $\phi^{-i}(\xi)$ to zero, where $i \geqslant 0$. This procedure yields a system of equations which can be solved to find α_k, $\phi(\xi)$ and $\phi'(\xi)$. Then the substitution of the values of α_k, $\phi(\xi)$ and $\phi'(\xi)$ into Eq. (2.4) completes the determination of exact solutions of Eq. (2.1).

3. Applications

3.1. The Drinfel'd–Sokolov–Wilson equation:

Now we will bring to bear the MSE method to find exact solutions and then the solitary wave solutions to the DSW equation in the form

$$\begin{aligned} &u_t + pvv_x = 0, \\ &v_t + qv_{xxx} + ruv_x + su_x v = 0, \end{aligned} \tag{3.1}$$

where p, q, r and s are nonzero parameters.

Now let us suppose that the traveling wave transformation equation be

$$u(\xi) = u(x,t), \quad v(\xi) = v(x,t), \quad \xi = x + \omega t. \tag{3.2}$$

The Eq. (3.2) reduces Eq. (3.1) into the following ODEs

$$\omega u' + pvv' = 0, \tag{3.3}$$

$$\omega v' + qv''' + ruv' + su'v = 0. \tag{3.4}$$

By integrating Eq. (3.3) with respect to ξ, and neglecting the constant of integration, we obtain

$$u = -\frac{pv^2}{2\omega}. \tag{3.5}$$

Substituting Eq. (3.5) into Eq. (3.4), we obtain

$$2q\omega v''' + 2\omega^2 v' - p(r+2s)v^2 v' = 0. \tag{3.6}$$

Integrating Eq. (3.6) with respect to ξ choosing constant of integration to zero, we obtain

$$2q\omega v'' + 2\omega^2 v - \frac{p(r+2s)v^3}{3} = 0. \tag{3.7}$$

Balancing the highest order derivative v'' and nonlinear term v^3 from Eq. (3.7), we obtain $3n = n + 2$, which gives $n = 1$.

Now for $n = 1$, using Eq. (2.4) we can write

$$v(\xi) = \alpha_0 + \alpha_1 \left(\frac{\phi'(\xi)}{\phi(\xi)}\right), \tag{3.8}$$

where α_0 and α_1 are constants to be determined such that $\alpha_1 \neq 0$, while $\phi(\xi)$ is an unknown function to be determined. It is easy to see that

$$v'' = \alpha_1\left(\frac{\phi'''}{\phi}\right) - 3\alpha_1\left(\frac{\phi''\phi'}{\phi^2}\right) + 2\alpha_1\left(\frac{\phi'}{\phi}\right)^3, \tag{3.9}$$

$$v^3 = \alpha_1^3\left(\frac{\phi'}{\phi}\right)^3 + 3\alpha_1^2\alpha_0\left(\frac{\phi'}{\phi}\right)^2 + 3\alpha_1\alpha_0^2\left(\frac{\phi'}{\phi}\right) + \alpha_0^3. \tag{3.10}$$

Now substituting the values of v, v'', v^3 in Eq. (3.7) and then equating the coefficients of ϕ^0, ϕ^{-1}, ϕ^{-2}, ϕ^{-3} to zero, we respectively obtain

$$-\frac{1}{3}pr\alpha_0^3 - \frac{2}{3}ps\alpha_0^3 + 2\omega^2\alpha_0 = 0, \tag{3.11}$$

$$2\omega q\phi''' + \left(2\omega^2 - pr\alpha_0^2 - 2ps\alpha_0^2\right)\phi' = 0, \tag{3.12}$$

$$6\omega q\phi'' + p(r+2s)\alpha_0\alpha_1\phi' = 0, \tag{3.13}$$

$$\left(-\frac{1}{3}pr\alpha_1^3 - \frac{2}{3}ps\alpha_1^3 + 4\omega q\alpha_1\right)(\phi')^3 = 0. \tag{3.14}$$

Solving Eq. (3.11), we get