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REVIEW PAPER

On fuzzy almost continuous convergence in fuzzy function spaces

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Abstract In this paper, we study the fuzzy almost continuous convergence of fuzzy nets on the set $FAC(X, Y)$ of all fuzzy almost continuous functions of a fuzzy topological space X into another Y. Also, we introduce the notions of fuzzy splitting and fuzzy jointly continuous topologies on the set $FAC(X, Y)$ and study some of its basic properties.

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Contents

1. Introduction and preliminaries

Throughout this paper X and Y mean fuzzy topological spaces (fts, for short). The concepts of fuzzy points [1], quasi-conicidence [2] and fuzzy nets [3] have proven to be suitable notions for several extensions.

Let X be a set. A fuzzy subset A of X is characterized by a membership function A: $X \to I$, where $I = [0, 1]$. The set of all fuzzy subsets of X will be denoted by I^X.

Let A be a fuzzy subset of X. The fuzzy set A', where $A'(x) = 1 - A(x)$, for every $x \in X$, is called the complement of A.

Let f: $X \to Y$ be a map. Then we have that:

(i) For a fuzzy subset B of Y, $f^{-1}(B)$ is defined as follows: $f^{-1}(B)(x) = B(f(x)), \forall x \in X$,
(ii) For a fuzzy subset A of X, $f(A)$ is defined as follows:

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$$f(A)(y) = \begin{cases} \sup_{x \in X}\{A(x)|f(x) = y\} & \text{if } f^{-1}(y) \neq \phi \\ 0 & \text{if } f^{-1}(y) = \phi. \end{cases}$$

Let X, Y and Z be a fuzzy topological spaces and F: $X \times Y \to Z$ be a map, then by F_{x_t}, where x_t is a fuzzy point in X, we denote the fuzzy continuous map of Y into Z, for which $F_{x_t}(y_r) = F(x_t, y_r)$, for every fuzzy point y_r in Y. Also, by $\widehat{F}$ we denote the map of X into the set $FAC(Y, Z)$ for which $\widehat{F}(x_t) = F_{x_t}$, for every x_t in X. Let G be a map of the space X into the set $FAC(Y, Z)$. By $\widetilde{G}$ we denote the map of the space $X \times Y$ into the space Z, for which $\widetilde{G}(x_t, y_r) = \widehat{G}(x_t)(y_r)$ for every fuzzy point x_t in X and y_r in Y.

If $\Im$ is a fuzzy topology on the set $FAC(X, Y)$, then $FAC_{\Im}(X, Y)$ is called a fuzzy function space.

For any two fuzzy topological spaces X and Y, the map $e : FAC_{\Im}(X, Y) \times X \to Y$; $(f, x_t) \mapsto f(x_t)$ is called a fuzzy evaluation map.

Definition 1.1 [1]. A fuzzy point x_t in X is a fuzzy set defined as follows:

$$x_t(y) = \begin{cases} t & \text{if } y = x \\ 0 & \text{otherwise} \end{cases}$$

where, $0 < t \leqslant 1$,t is called its value and x is its support. The set of all fuzzy points in X is denoted by $Pt(X)$.

Definition 1.2 [4]. A fuzzy topology $\mathfrak{F}$ on X is a family of fuzzy subsets of X such that:

(i) $\mathfrak{F}$ contains all constant fuzzy subsets of X,
(ii) $A \cap B \in \mathfrak{F}$, for each $A, B \in \mathfrak{F}$,
(iii) If $\{A_\lambda\}_{\lambda \in \Lambda}$ is a subfamily of $\mathfrak{F}$, then $\cup_{\lambda \in \Lambda} A_\lambda \in \mathfrak{F}$.

The pair $(X, \mathfrak{F})$ is called a fuzzy topological space denoted by fts [4].

Definition 1.3 [2]. A fuzzy subset A of X is called a neighborhood (or nbd) of a fuzzy point x_t iff there exists a fuzzy open set V in X such that $x_t \in V \subseteq A$. Also, x_t is called quasi-coincident with A, denoted by $x_t q\ A$ if $t + A(x) > 1$. A is called a quasi-neighborhood, denoted by Q-nbd of x_t, if there exists a fuzzy open set V in X such that $V \subseteq A$ and $x_t q\ V$.

Definition 1.4 [5]. Let f be a map of X into Y. Then f is called fuzzy almost continuous at $x_t \in Pt(X)$ iff for every fuzzy open nbd V of $f(x_t)$ there exists a fuzzy open nbd U of x_t such that $f(U) \subseteq int(cl(V))$.

Definition 1.5 [6]. A fuzzy subset V of a fuzzy topological space (X, τ_X) is called a fuzzy regular open if $int(cl(V)) = V$ and V is regular closed if $V = cl(int(V))$.

Definition 1.6 [6,9]. A mapping f: $(X, \tau_X) \to (Y, \tau_Y)$ from an fts X into another Y is called fuzzy almost continuous if $f^{-1}(U) \in \tau_X$, for each fuzzy regular open subset U of Y.

Definition 1.7 [3]. Let $(D, \leqslant)$ be a directed set, X be a set and $Pt(X)$ be the set of all fuzzy points in X. The function S: $D \to Pt(X)$ is called a fuzzy net in X denoted by $\{S(n)$: $n \in D\}$ or $\{S_n$: $n \in D\}$.

Definition 1.8 [7]. A fuzzy net $\{S(n)$: $n \in D\}$ in a fuzzy topological space X is said to be fuzzy weak θ-converges to x_t in X if for every fuzzy open nbd V of x_t there is some $n_0 \in D$ such that $S(n) \in int(cl(V))$, for every $n \in D$, $n \geqslant n_0$.

Definition 1.9 [8]. Let $(X, \mathfrak{F})$ be an fts. A fuzzy subset U is called RQ-nbd of a fuzzy point $x_t \in Pt(X)$ iff there exists a fuzzy regular open set V in X such that $x_t q\ V \subseteq U$.

Definition 1.10 [8]. A fuzzy point x_t is said to be fuzzy δ-cluster point of a fuzzy subset V of an fts X iff each RQ-nbd of x_t is quasi-coincident with V. The union of all fuzzy δ-cluster points of V is defined to be δ-closure of V and denoted by $\delta - cl(V)$. If $V = \delta - cl(V)$, then V is called δ-closed fuzzy set.

Throughout this paper $FAC(X, Y)$ denotes the set of all fuzzy almost continuous maps of X into Y.

2. Fuzzy almost continuous functions

Theorem 2.1. *A map f of a space X into a space Y is fuzzy almost continuous at $x_t \in Pt(X)$ iff for every fuzzy net $\{S(n)$: $n \in D\}$ in X which fuzzy converges to x_t, we have that the fuzzy net $\{f(S(n))$: $n \in D\}$ in Y fuzzy weakly θ-converges to $f(x_t)$ in Y.*

Proof. Suppose that f is fuzzy almost continuous at $x_t \in Pt(X)$ and let $\{S(n)$: $n \in D\}$ be a fuzzy net in X fuzzy converges to x_t. Then, for every fuzzy open nbd V of $f(x_t)$ in Y there exists a fuzzy open nbd U of x_t in X such that $f(U) \subseteq int(cl(V))$. Then, there exists an element $n_0 \in D$ such that $S(n) \in V$, for every $n \in D$, $n \geqslant n_0$. Thus, $f(S(n)) \in int(cl(V))$, for every $n \geqslant n_0$, $n \in D$. Therefore, the fuzzy net $\{f(S(n))$: $n \in D\}$ in Y fuzzy weakly θ-converges to $f(x_t)$.

Conversely, if the map f is not fuzzy almost continuous at $x_t \in Pt(X)$, then for some fuzzy open nbd V of $f(x_t)$, $f(U) \not\subseteq int(cl(V))$, for every fuzzy open nbd U of x_t in X. Thus, for every fuzzy open nbd U of x_t we can find x_t^U such that $f(x_t^U) \notin int(cl(V))$. Let $N(x_t)$ be the set of all fuzzy open nbds of x_t in X. The set $N(x_t)$ with the relation $\{U_1 \leqslant U_2$ iff $U_2 \subseteq U_1\}$ form a directed set. Clearly, the fuzzy net $\{x_t^U : U \in N(x_t)\}$ fuzzy converges to x_t in X but the fuzzy net $\{f(x_t^U) : U \in N(x_t)\}$ does not fuzzy weakly θ-converges to $f(x_t)$ in Y. Hence, the map f is fuzzy almost continuous at $x_t \in Pt(X)$. □

Definition 2.1. A fuzzy net $\{f_m$: $m \in D\}$ in $FAC(X, Y)$ fuzzy almost continuously converges to $f \in FAC(X, Y)$ iff for every fuzzy net $\{S(n)$: $n \in \wedge\}$ in X which fuzzy converges to $x_t \in Pt(X)$ we have that the fuzzy net $\{f_m(S(n))$: $(n, m) \in \wedge \times D\}$ fuzzy weakly θ-converges to $f(x_t)$ in Y.

Theorem 2.2. *A fuzzy net $\{f_m$: $m \in D\}$ in $FAC(X, Y)$ fuzzy almost continuously converges to $f \in FAC(X, Y)$ iff for every fuzzy point $x_t \in X$ and for every fuzzy open nbd V of $f(x_t)$ in Y there exist an element $m_0 \in D$ and a fuzzy open nbd U of x_t in X such that $f_m(U) \subseteq int(cl(V))$, for every $m \in D$, $m \geqslant m_0$.*