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A General scheme for dithering multidimensional signals, and a visual instance of encoding images with limited palettes

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Abstract The core contribution of this paper is to introduce a general neat scheme based on soft vector clustering for the dithering of multidimensional signals that works in any space of arbitrary dimensionality, on arbitrary number and distribution of quantization centroids, and with a computable and controllable quantization noise. Dithering upon the digitization of one-dimensional and multi-dimensional signals disperses the quantization noise over the frequency domain which renders it less perceptible by signal processing systems including the human cognitive ones, so it has a very beneficial impact on vital domains such as communications, control, machine-learning, etc. Our extensive surveys have concluded that the published literature is missing such a neat dithering scheme. It is very desirable and insightful to visualize the behavior of our multidimensional dithering scheme; especially the dispersion of quantization noise over the frequency domain. In general, such visualization would be quite hard to achieve and perceive by the reader unless the

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target multidimensional signal itself is directly perceivable by humans. So, we chose to apply our multidimensional dithering scheme upon encoding true-color images – that are 3D signals – with palettes of limited sets of colors to show how it minimizes the visual distortions – esp. contouring effect – in the encoded images.

1. Introduction

The main contribution of this paper is to introduce a general neat scheme for the *dithering* of multidimensional signals that is able to deal with arbitrary dimensionality, arbitrary number and distribution of quantization centroids, and with computable and controllable noise power. In order to proceed with presenting this novel multidimensional dithering scheme, it is necessary first to formally review one-dimensional signal digitization, quantization noise, and dithering.

The digitization of an analog one dimensional signal – known as *Analog-to-Digital* ("A-to-D" or "A2D") conversion – aims at mapping any given sample of the signal within its dynamic range $q_{\min} \leqslant q \leqslant q_{\max}$ to one element of a pre-defined set of quantum levels $\{c_1, c_2, \ldots, c_i, \ldots, c_L\}$; $q_{\min} \leqslant c_i \leqslant q_{\max}$, $L \geqslant 2$. In order to minimize the digitization error, this mapping is typically done through the *minimum-distance* criterion; i.e. the signal sample is mapped to the nearest quantum level, which can be formulated as follows:

$$q \xrightarrow{A\text{-}to\text{-}D} i_0 : i_0 = \arg \min_{\forall k; 1 \le k \le L} \{d(q, c_k)\}, \tag{1}$$

where $d(q_1, q_2)$ is any legitimate distance criterion between $q_1, q_2 \in \Re^1$. The digitization of a given signal sample in the 1D space is reduced into a simple selection of one of – at most – the two quantum levels enclosing that signal sample (Roberts, 2007; Widrow and Kollár, 2008) as illustrated by Fig. 1 below.

The sum of the squared digitization errors of all the emerging signal samples make the quantization noise which is formulated as follows (Roberts, 2007; Widrow and Kollár, 2008):

$$E_q^2 = \sum\nolimits_{\forall q} e_q^2(q) = \sum\nolimits_{\forall q} (q - c_{i_0})^2. \tag{2}$$

The distribution of the set of quantum levels over the dynamic range of the signal may be regular that $c_i = q_{\min} + (i-1) \cdot \frac{q_{\max} - q_{\min}}{L}$ and is then called *regular quantization*. When the distribution of emerging signal samples to be digitized is significantly irregular, the distribution of the quantum levels may be designed to track that irregular one of emerging samples, and is then called *adaptive quantization*.[5] Adaptive quantization aims at minimizing the quantization noise for any given number L of quantum levels (Roberts, 2007; Widrow and Kollár, 2008).

Increasing L obviously decreases both the digitization errors and quantization noise; however, there are hardware and/or computational cost limitations on the size of L to be deployed in a given digitization scheme. When L is not large enough to adequately capture the resolution of the analog signal, the digitized signal suffers from obtrusive artifacts that render its information content into a significantly distorted version from that carried by the original analog signal. This may turn into a serious drop of quality if the digitized signal is destined for human perception; e.g. digital audio, or may turn into a serious source of error if the digitized signal is forwarded to some further processing; e.g., machine learning, control systems, etc.

For example, consider an audio signal of a single tone – i.e. a purely sinusoidal wave – at 500 Hz. In the frequency domain, this analog signal shows a single impulse at 500 Hz and nothing elsewhere. When, this audio signal is digitized via 16-bit quantization; i.e. $2^{16} = 65{,}536$ quantum levels, the resulting digital signal in the frequency domain seems (almost) the same as the original analog one as illustrated by the blue curve at the top of Fig. 2. On the other hand, when the same audio signal is digitized via 6-bit quantization; i.e. $2^6 = 64$ quantum levels, the resulting digital signal in the frequency domain shows a major peak at 500 Hz but also other considerable harmonics like the one around 4500 Hz as illustrated by the red curve at the middle of Fig. 2. These obtrusive harmonics mean that the digitized signal is not corresponding any more to a pure single tone, but is corresponding to a composite one where irritating false whistles are superimposed on the original pure tone (Pohlmann, 2005).

Researchers and engineers had realized since decades that this problem is caused by the concentration of the digitization errors within narrow bands of the signal, and has accordingly realized that dispersing the digitization errors over wider bands in the frequency domain would produce a better digitized signal where obtrusive artifacts are less conspicuous. With signals digitized this way, humans would perceive a better quality, and digital signal processing systems would perform more robustly. Dispersing the digitization errors over wider bands is typically achieved through adding controlled noise to the analog signal just before the A-to-D conversion (Petri, 1996; Schuchman, 1964). This process is popularly known as *"dithering"* whose simplest – and also most commonly used – variant adds to each analog signal sample q some ± random value whose amplitude is half the distance between the two enclosing quantum levels c_i and c_{i-1}. Digitization with this kind of dithering may be formulated as follows:

$$i_0^* = \underset{\forall k: i-1 \le k \le i}{\operatorname{argmin}} \left\{ d\left(q + rand\left(\frac{c_{i-1} - c_i}{2}, \frac{c_i - c_{i-1}}{2} \right), c_k \right) \right\}. \tag{3}$$

Digitization with dithering of a given signal sample as described by Eq. (3) is still a selection of one of the two quantum levels enclosing that signal sample; however, unlike Eq. (1) this selection is a stochastic process where the chances of attributing the sample to each of the two quantum levels are given by:

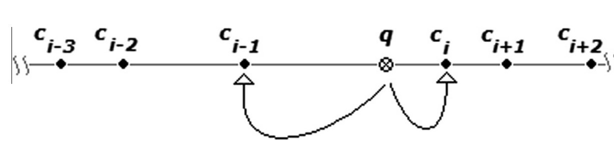

Figure 1 Dithering in 1D space; *only the two* enclosing quantum levels compete for the given point.

[5] The distribution of the quantum levels in Fig. 1 is assumed to belong to this second kind of quantization.