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ORIGINAL ARTICLE

Solitary wave solutions Zakharov–Kuznetsov–Benjamin–Bona–Mahony (ZK–BBM) equation

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Abstract In this paper, exp-function method is used to construct generalized solitary solutions of the Zakharov–Kuznetsov–Benjamin–Bona–Mahony equation. It is shown that the exp-function method, with the help of symbolic computation, provides a straightforward and powerful mathematical tool to solve such nonlinear equations.The performance of the method is reliable, efficient and it gives useful exact solutions.

MATHEMATICS SUBJECT CLASSIFICATION: 35Cxx; 35Dxx; 35Qxx

1. Introduction

The nonlinear partial differential equations play a pivotal role in the mathematical modeling of diversified physical phenomena. Finding exact solutions [1–27] of nonlinear evolution equations (NLEEs) has become one of the most exciting and extremely active areas of research investigation. The investigation of exact travelling wave solutions to nonlinear evolution equations plays a vital role in the study of nonlinear physical phenomena. The wave phenomena are observed in fluid dynamics, plasma, elastic media, optical fibers, etc. Many effective methods have been presented such as variational iteration method [1], homotopy perturbation method [2], Adomian's decomposition method [3] and others [4]. The aim of the present paper was to extend the exp-function method to find new solitary solutions and periodic solutions for Zakharov–Kuznetsov–Benjamin–Bona–Mahony equation. Recently, Shakeel and Mohyud-Din [5] used the (G'/G)-expansion method to obtain solutions of Zakharov–Kuznetsov–Benjamin–Bona–Mahony equation.

2. Exp-function method

Consider the general nonlinear partial differential equation of the type

$$P(u, u_t, u_x, u_{tt}, u_{xx}, u_{xxx}, \ldots) = 0 \tag{1}$$

Using a transformation

$$\eta = kx + \omega t, \tag{2}$$

where k and ω are constants, we can rewrite Eq. (1) in the following nonlinear ODE,

$$Q(u, u', u'', u''', u'''', \ldots) = 0 \tag{3}$$

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where the prime denotes derivative with respect to η.

According to the exp-function method, which was developed by He and Wu, we assume that the wave solutions can be expressed in the following form

$$u(\eta) = \frac{\sum_{n=-c}^{d} a_n exp(n\eta)}{\sum_{m=-p}^{q} b_m exp(m\eta)} \tag{4}$$

where p, q, c and d are positive integers which are known to be further determined, a_n and b_m are unknown constants. We can rewrite Eq. (4) in the following equivalent form

$$u(\eta) = \frac{a_c exp(c\eta) + \cdots + a_{-d} exp(-d\eta)}{b_p exp(p\eta) + \cdots + b_{-q} exp(-q\eta)} \tag{5}$$

To determine the value of c and p, we balance the linear term of highest order of Eq. (4) with the highest order nonlinear term. Similarly, to determine the value of d and q, we balance the linear term of lowest order of Eq. (3) with lowest order nonlinear term.

3. Solution procedure

3.1. Zakharov–Kuznetsov–Benjamin–Bona–Mahony equation

Consider the following Zakharov–Kuznetsov–Benjamin–Bona–Mahony (ZK–BBM) equation

$$u_t + u_x - 2auu_x - bu_{xxt} = 0. \tag{6}$$

Introducing a transformation as $\eta = kx + \omega t$ we can convert Eq. (6) into ordinary differential equations

$$\omega u' + ku' - 2akuu' - bk^2\omega u''' = 0, \tag{7}$$

where the prime denotes the derivative with respect to η.The trial solution of the Eq. (7) can be expressed as follows,

$$u(\eta) = \frac{a_c exp(c\eta) + \cdots + a_{-d} exp(-d\eta)}{b_p exp(p\eta) + \cdots + b_{-q} exp(-q\eta)}.$$

To determine the value of c and p we balance the linear term of highest order of Eq. (7) with the highest order nonlinear term and to determine the value of d and q we balance the linear term of lowest order of Eq. (7) with the lowest order nonlinear term. We obtain $p = c$ and $d = q$.

3.1.1. Case 3.1.1

we can freely choose the values of c and$p$, we balance the linear term of highest order of Eq. (7) with the highest order nonlinear term, but we will illustrate that the final solution does not strongly depend upon the choice of values of c and d. For simplicity, we set $p = c = 1$ and $d = q = 1$ Eq. (5) reduces to

$$u(\eta) = \frac{a_1 exp(\eta) + a_0 + a_{-1} exp(-\eta)}{b_1 exp(\eta) + a_0 + b_{-1} exp(-\eta)} \tag{8}$$

Substituting Eq. (8) into Eq. (7), we have

$$\begin{aligned} C_1 = {} & \omega a_1 b_0^2 + k a_1 b_0^2 - \alpha k a_1^2 b_{-1} + 2\omega a_0 b_1 b_0 + 2k a_0 b_1 b_0 \\ & - \alpha k a_0^2 b_1 + k a_{-1} b_1^2 + \omega a_{-1} b_1^2 - 2\alpha k a_{-1} a_1 b_1 - 4\beta\omega k^2 a_{-1} b_1^2 \\ & + \beta\omega k^2 a_0 b_1 b_0 - 2\alpha k a_0 a_1 b_0 - \beta\omega k^2 a_1 b_0^2 + 4\beta\omega k^2 a_1 b_1 b_{-1} \\ & + 2\omega a_1 b_1 b_{-1} + 2k a_1 b_1 b_{-1} \end{aligned}$$

$$\begin{aligned} C_{-1} = {} & \omega a_{-1} b_0^2 + k a_{-1} b_0^2 - \alpha k a_{-1}^2 b_{-1} + 2\omega a_0 b_{-1} b_0 + 2k a_0 b_{-1} b_0 \\ & - \alpha k a_0^2 b_1 + k a_1 b_{-1}^2 + \omega a_1 b_{-1}^2 - 2\alpha k a_{-1} a_1 b_{-1} \\ & - 4\beta\omega k^2 a_1 b_{-1}^2 + \beta\omega k^2 a_0 b_{-1} b_0 - 2\alpha k a_0 a_{-1} b_0 \\ & - \beta\omega k^2 a_{-1} b_0^2 + 4\beta\omega k^2 a_{-1} b_1 b_{-1} + 2\omega a_{-1} b_1 b_{-1} \\ & + 2k a_{-1} b_1 b_{-1} \end{aligned}$$

$$\begin{aligned} C_2 = {} & -\alpha k a_1^2 b_0 + 2\omega a_1 b_1 b_0 + 2k a_1 b_1 b_0 + k a_0 b_1^2 + \omega a_0 b_1^2 \\ & - \beta\omega k^2 a_0 b_1^2 + \beta\omega k^2 a_1 b_1 b_0 - 2\alpha k a_0 a_1 b_1 \end{aligned}$$

$$\begin{aligned} C_{-2} - {} & \alpha k a_{-1}^2 b_0 + 2\omega a_{-1} b_{-1} b_0 + 2k a_{-1} b_{-1} b_0 + k a_0 b_{-1}^2 + \omega a_0 b_{-1}^2 \\ & - \beta\omega k^2 a_0 b_{-1}^2 + \beta\omega k^2 a_{-1} b_{-1} b_0 - 2\alpha k a_0 a_{-1} b_{-1} \end{aligned}$$

$$C_3 = -\alpha k a_1^2 b_1 + k a_1 b_1^2 + \omega a_1 b_1^2$$

$$C_{-3} = -\alpha k a_{-1}^2 b_{-1} + k a_{-1} b_{-1}^2 + \omega a_{-1} b_{-1}^2$$

$$\begin{aligned} C_0 = {} & -\alpha k a_0^2 b_0 + 2\omega a_{-1} b_1 b_0 + 2\omega a_0 b_1 b_{-1} + 2\omega a_1 b_{-1} b_0 \\ & + 2k a_{-1} b_1 b_0 + 2k a_0 b_1 b_{-1} + 2k a_1 b_{-1} b_0 + k a_0 b_0^2 \\ & + \omega a_0 b_0^2 - 3\beta\omega k^2 a_1 b_{-1} b_0 - 3\beta\omega k^2 a_{-1} b_1 b_0 \\ & + 6\beta\omega k^2 a_0 b_1 b_{-1} - 2\alpha k a_0 a_{-1} b_1 - 2\alpha k a_1 a_{-1} b_0 \\ & - 2\alpha k a_0 a_1 b_{-1} \end{aligned} \tag{9}$$

are constants obtained by Maple 15. Equating the coefficients of $\exp(n\eta)$ to be zero, we obtain

$$\{C_3 = 0 \; C_2 = 0, \quad C_1 = 0, \quad C_{-3} = 0 \; C_{-2} = 0, \quad C_{-1} = 0, \quad C_0 = 0\} \tag{10}$$

Solution of (10) we have following solution sets satisfy the given equation,

3.1.1.1. 1st Solution set.

$$\left\{ \begin{array}{l} \omega = \frac{k}{-1+k^2}, \quad a_{-1} = 0, \quad a_0 = a_0, \quad a_1 = 0, \quad b_{-1} = \frac{1}{36}\frac{a_0^2(1-2k^2+k^4)}{k^4 b_1}, \\ b_0 = \frac{1}{3}\frac{a_0(-1+k^2)}{k^2}, \quad b_1 = b_1 \end{array} \right\} \tag{11}$$

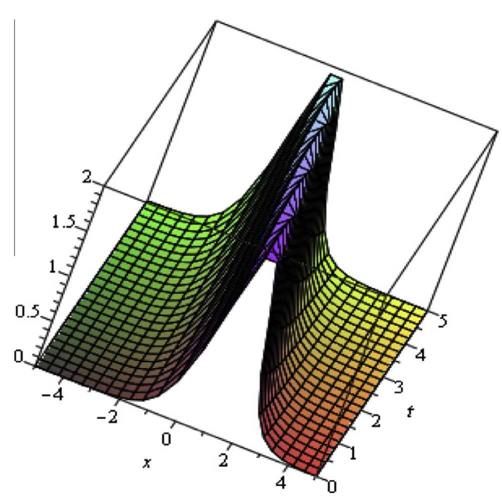

Figure 3.1a Singular Kink wave solution of Eq. (12).