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ORIGINAL ARTICLE

Wave propagation in a transversely isotropic magneto-electro-elastic solid bar immersed in an inviscid fluid

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Abstract Wave propagation in a transversely isotropic magneto-electro-elastic solid bar immersed in an inviscid fluid is discussed within the frame work of linearized three dimensional theory of elasticity. Three displacement potential functions are introduced to uncouple the equations of motion, electric and magnetic induction. The frequency equations that include the interaction between the solid bar and fluid are obtained by the perfect slip boundary conditions using the Bessel functions. The numerical calculations are carried out for the non-dimensional frequency, phase velocity and attenuation coefficient by fixing wave number and are plotted as the dispersion curves. The results reveal that the proposed method is very effective and simple and can be applied to other bar of different cross section by using proper geometric relation.

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1. Introduction

The smart composite material such as a magneto-electro-elastic material exhibits the desirable coupling effect between electric and magnetic fields and has gained considerable importance since last decade. These materials have the capacity to convert one form of energy namely, magnetic, electric and mechanical energy to another form of energy. The composite consisting of piezoelectric and piezomagnetic components has found increasing application in engineering structures, particularly in smart/intelligent structure system. In addition, magnetoelectroelastic materials have been used extensively in the design of light weighted and high performance sensors and transducers due to direct and converse piezoelectricity effects. The direct piezoelectric effect is used in sensing applications, such as in force or displacement sensors. The converse piezoelectric effects are used in transduction applications, such as in motors and device that precisely control positioning, and in generating sonic and ultrasonic signals. This study may be used in applications involving nondestructive testing (NDT), qualitative

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nondestructive evaluation (QNDE) of large diameter pipes and health monitoring of other ailing infrastructures in addition to check and verify the validity of FEM and BEM for such problems.

Pan [1] and Pan and Heyliger [2] have discussed the three-dimensional behavior of magnetoelectroelastic laminates under simple support. An exact solution for magnetoelectro-elastic laminates in cylindrical bending has also been obtained by Pan and Heyliger [3]. Pan and Han [4] derived the exact solution for functionally graded and layered magneto-electro-elastic plates. Feng and Pan [5] discussed the dynamic fracture behavior of an internal interfacial crack between two dissimilar magneto-electro-elastic plates. Buchanan [6] developed the free vibration of an infinite magneto-electro-elastic cylinder. Dai and Wang [7,8] have studied thermo-electro-elastic transient responses in piezoelectric hollow structures and hollow cylinder subjected to complex loadings.

Later Wang with Kong et al. [9] presented the thermo-magneto-dynamic stresses and perturbation of magnetic field vector in a non-homogeneous hollow cylinder. Annigeri et al. [10–12] studied respectively, the free vibration of clamped-clamped magneto-electro-elastic cylindrical shells, free vibration behavior of multiphase and layered magneto-electro-elastic beam, free vibrations of simply supported layered and multi-phase magneto-electro-elastic cylindrical shells. Hon et al. [13] analyzed a point heat source on the surface of semi-infinite transversely isotropic electro-magneto-thermo-elastic materials. Sharma and Mohinder Pal [14] developed the Rayleigh-Lamb waves in magneto-thermo-elastic homogeneous isotropic plate. Later Sharma and Thakur [15] studied the effect of rotation on Rayleigh-Lamb waves in magneto-thermo-elastic media. Gao and Noda [16] presented the thermal-induced interfacial cracking of magnetoelectroelastic materials. Bin et al. [17] studied the wave propagation in non-homogeneous magneto-electo-elastic plates.

Sinha et al. [18] made an investigation about the axisymmetric wave propagation in circular cylindrical shell immersed in a fluid, in two parts. In Part I, the theoretical analysis of the propagation modes is discussed and in Part II, the axisymmetric modes excluding tensional modes are obtained both theoretically and experimentally and are compared. Berliner and Solecki [19] investigated wave propagation in a fluid loaded transversely isotropic cylinder. In that paper, Part I consists of the analytical formulation of the frequency equation of the coupled system consisting of the cylinder with inner and outer fluid and Part II gives the numerical results.

Ponnusamy [20] has studied the wave propagation in a generalized thermoelastic cylinder of arbitrary cross-section immersed in a fluid using the Fourier expansion collocation method. Recently, Ponnusamy and Selvamani [21,22] have studied respectively, the three dimensional wave propagation of transversely isotropic magneto thermo elastic and generalized thermo elastic cylindrical panel in the context of the linear theory of thermo elasticity.

In this problem, the wave propagation in a transversely isotropic magneto-electro-elastic solid bar immersed in an inviscid fluid is studied using Bessel function. Three displacement potential functions, electric field vector and magnetic fields are used to uncouple the equations of motion. The frequency equations are obtained from the perfect slip boundary conditions. The computed non-dimensional frequencies, phase velocity and attenuation coefficient are plotted in the form of dispersion curves and their characteristics are discussed.

2. Formulation of the problem

The constitutive equations of a transversely isotropic linear magneto-electro-elastic material, involving stresses σ_j, strain S_{ij}, electric displacements D_{ij}, electric field E_k, magnetic induction B_j and magnetic field H_k are considered in the lines of Buchannan [6],

$$\sigma_j = C_{jk}S_k - e_{jk}E_k - q_{kj}H_k \tag{1}$$

$$D_j = e_{jk}S_k + \varepsilon_{jk}E_k + m_{jk}H_k \tag{2}$$

$$B_j = q_{jk}S_k + m_{jk}E_k + \mu_{jk}H_k \tag{3}$$

where C_{jk},ε_{jk} and μ_{jk} are the elastic, dielectric and magnetic permeability coefficients respectively; e_{kj}, q_{kj} and m_{jk} are the piezoelectric, piezomagnetic and magnetoelectric material coefficients.

The strain S_{ij} is related to the displacements corresponding to the cylindrical coordinates (r, θ, z) which are given by

$$S_{rr} = \frac{\partial u_r}{\partial r}, \quad S_{\theta\theta} = \frac{1}{r}\left(u_r + \frac{\partial u_\theta}{\partial \theta}\right), \quad S_{zz} = \frac{\partial u_z}{\partial z}$$
$$S_{r\theta} = \left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right), S_{rz} = \left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right),$$
$$S_{\theta z} = \left(\frac{1}{r}\frac{\partial u_z}{\partial \theta} + \frac{\partial u_\theta}{\partial z}\right) \tag{4}$$

where u_r,u_θ and u_z are the mechanical displacements corresponding to the cylindrical coordinate directions r, θ and z. The relation between the electric field vector E_i and the electric potential ϕ is given by

$$E_r = -\frac{\partial \phi}{\partial r}, \quad E_\theta = -\frac{1}{r}\frac{\partial \phi}{\partial \theta} \text{ and } E_z = -\frac{\partial \phi}{\partial z} \tag{5}$$

Similarly, the magnetic field H_i is related to the magnetic potential ψ as

$$H_r = -\frac{\partial \psi}{\partial r}, \quad H_\theta = -\frac{1}{r}\frac{\partial \psi}{\partial \theta} \text{ and } H_z = -\frac{\partial \psi}{\partial z} \tag{6}$$

The basic governing equations of motion, electrostatic displacement D_j and magnetic induction B_j in cylindrical co-ordinates (r, θ, z) system, in the absence of volume force are

$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) = \rho\frac{\partial^2 u_r}{\partial t^2} \tag{7a}$$

$$\frac{\partial \sigma_{\theta r}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \sigma_{\theta z}}{\partial z} + \frac{2\sigma_{\theta r}}{r} = \rho\frac{\partial^2 u_\theta}{\partial t^2} \tag{7b}$$

$$\frac{\partial \sigma_{zr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{z\theta}}{\partial \theta} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{rz}}{r} = \rho\frac{\partial^2 u_z}{\partial t^2} \tag{7c}$$

$$\frac{\partial D_r}{\partial r} + \frac{D_r}{r} + \frac{\partial D_\theta}{\partial \theta} + \frac{\partial D_z}{\partial z} = 0 \tag{7d}$$

$$\frac{\partial B_r}{\partial r} + \frac{B_r}{r} + \frac{\partial B_\theta}{\partial \theta} + \frac{\partial B_z}{\partial z} = 0 \tag{7e}$$

where the stress strain relation for the transversely isotropic medium is given by