Journal of the Egyptian Mathematical Society (2016) 24, 116–124

Egyptian Mathematical Society

Journal of the Egyptian Mathematical Society

www.etms-eg.org
www.elsevier.com/locate/joems

ORIGINAL ARTICLE

Numerical study of shock waves in non-ideal magnetogasdynamics (MHD)

Addepalli Ramu *, Narsimhulu Dunna, Dipak Kumar Satpathi

Department of Mathematics, BITS-Pilani, Hyderabad Campus, India

Received 24 February 2014; revised 30 August 2014; accepted 21 October 2014
Available online 11 February 2015

KEYWORDS

Similarity solutions;
Shock waves;
Magnetogasdynamics;
Rankine–Hugoniot condition;
Mie–Gruneisen type;
Numerical solution

Abstract One-dimensional unsteady adiabatic flow of strong converging shock waves in cylindrical or spherical symmetry in MHD, which is propagating into plasma, is analyzed. The plasma is assumed to be non-ideal gas whose equation of state is of Mie–Gruneisen type. Suitable transformations reduce the governing equations into ordinary differential equations of Poincare type. In the present work, McQueen and Royce equations of state (EOS) have been considered with suitable material constants and the spherical and cylindrical cases are worked out in detail to investigate the behavior and the influence on the shock wave propagation by energy input and $\beta(\rho/\rho_0)$, the measure of shock strength. The similarity solution is valid for adiabatic flow as long as the counter pressure is neglected. The numerical technique applied in this paper provides a global solution to the implosion problem for the flow variables, the similarity exponent α for different Gruneisen parameters. It is shown that increasing $\beta(\rho/\rho_0)$ does not automatically decelerate the shock front but the velocity and pressure behind the shock front increases quickly in the presence of the magnetic field and decreases slowly and become constant. This becomes true whether the piston is accelerated, is moving at constant speed or is decelerated. These results are presented through the illustrative graphs and tables. The magnetic field effects on the flow variables through a medium and total energy under the influence of strong magnetic field are also presented.

MATHEMATICS SUBJECT CLASSIFICATION (MSC): 76M55; 76L05; 76W05

1. Introduction

Shock processes occur naturally in various astrophysical situations such as supernova explosions, photo-ionized gas, stellar winds, and collisions between high velocity clumps of interstellar gas. Magnetogasdynamics applies to many conductive fluid and plasma flows encountered in nature. In several circumstances, the flow is subject to a strong as well as a weak magnetic field. Such situation can be thought

* Corresponding author.
Peer review under responsibility of Egyptian Mathematical Society.

Production and hosting by Elsevier

http://dx.doi.org/10.1016/j.joems.2014.10.002
1110-256X © 2015 Production and hosting by Elsevier B.V. on behalf of Egyptian Mathematical Society.

of occurring in earth's liquid core, and is present in solar physics such as sunspots, solar flares, solar corona, and solar winds. The strong magnetic fields play significant roles in the dynamics of the interstellar medium. A theoretical study of the imploding shock wave near the center of convergence, in an ideal gas was first investigated by Guderley [1]. Several authors contributed to this investigation and we mention the contributions of, Hafner [2], Manganaro and Oliveri [3], Sharma and Radha [4], Hunter and Ali [5], Sharma and Arora [6], Stanyukovich [7], Chisnell [8], Lazarus and Richtmyer [9], Ramu and Ranga Rao [10], Madhumita and Sharma [11], Sen [12], who presented high accuracy results and alternative approaches for the investigation of implosion problem. The propagation of shock waves under the influence of strong magnetic field is of great interest to many researchers in various fields such as astrophysics, nuclear science, geophysics, and plasma physics. MHD shock waves in perfect gas are under extensive exploration and attained good attention in the past decades. Propagation of shock waves in magneto hydrodynamics (MHD) has been studied by several researchers. De Hoffmann and Teller [13] developed a mathematical treatment for the motion of MHD shock waves in the very weak and very strong magnetic fields. Bazer and Ericson [14] were first among the many researchers to study the hydromagnetic shocks for astrophysical applications and analytical solutions were presented by Genot [15] for anisotropic MHD shocks. A number of approaches namely, the similarity method, power series solution method, CCW method have been used for the theoretical investigations of MHD shock waves in homogeneous and inhomogeneous media.

In the recent years much attention has been focused on the self-similar solutions using similarity transformations because of their wide applications in determining solutions of nonlinear differential equations of physical interest. The gas attains very high temperature due to the propagation of shock waves and at such a high temperature, the gas gets ionized, hence effects of magnetic field become significant in the study of converging shock waves. The study of MHD shock waves in a non-ideal gas is of great scientific interest in many problems because of their applications in the areas of astrophysics, oceanography, atmospheric sciences, hypersonic aerodynamics and hypervelocity impact.

In this paper a model to determine the similarity solutions to the problem of gas dynamic flow under the influence of strong magnetic field is presented. The problem treated here involves distinct features: the global behavior of the physical parameter has been studied; the initial pressure ratio is confined to a moderate value. The path of the piston is imposed as boundary condition. Thus an accelerated, a decelerated or a constant velocity piston can be specified. Self-similarity requires the velocity of shock and the velocity of piston to be proportional to some power law $R(t) \propto (t)^{\alpha}$ where $R(t)$ is the position of the shock wave front from the center at time t and α is the similarity exponent. The numerical values of similarity exponents and profiles of flow variables are obtained. These are presented through the illustrative graphs and tables. The magnetic field effects on the flow variables through a medium and total energy under the influence of strong magnetic field are also presented.

2. Basic equations and boundary conditions

The non-steady one dimensional flow is a function of two independent variables the time t and the space coordinate r. The conservation equations governing the flow can be written as [12,16–20]

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{(m-1)\rho u}{r} = 0 \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \rho^{-1}\left(\frac{\partial p}{\partial r} + \frac{\partial h}{\partial r}\right) = 0 \tag{2}$$

$$\frac{\partial p}{\partial t} + u\frac{\partial p}{\partial r} - a^2\left(\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right) = 0 \tag{3}$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + 2h\frac{\partial u}{\partial r} + 2h(m-1)u/r = 0 \tag{4}$$

where $\rho(r,t)$, $u(r,t)$ and $p(r,t)$ denote the density, velocity, and pressure of the gas particles behind the shock front, $h(r,t)$ is the magnetic pressure defined by $h = \frac{\mu H^2}{2}$ with μ as magnetic permeability and H is the transverse magnetic field, $a^2 = (\Gamma + 1)p/\rho$ is the equilibrium speed of sound, Γ is the Gruneisen coefficient, $m = 2(3)$ denote shock wave in cylindrical (spherical) geometry.

It is assumed that the plasma has infinite electrical conductivity and permeated by an axial magnetic field orthogonal to the trajectories of the gas particles. Shock is assumed to be strong and propagating into a medium according to a power law $R(t) \propto (t)^{\alpha}$, where $R(t)$ is the position of the shock wave front from the center at time t and $t = 0$ corresponds to the instant of the convergence when $R = 0$. The equation of state under equilibrium condition is of Mie–Gruneisen type [10],

$$\mathrm{p} = \rho e \Gamma(\rho/\rho_0) \tag{5}$$

where the function $\Gamma(\rho/\rho_0)$ is the Gruneisen parameter.

2.1. Boundary conditions

The boundary conditions at shock front due to Rankine–Hugoniot, can be written as [7,18,20]

$$\rho = \frac{\Gamma+2}{\Gamma}\rho_0\left(1 + \frac{2}{\Gamma}\left(\frac{\mathrm{a}_0}{\mathrm{D}}\right)^2\right) \quad u = \frac{2}{\Gamma+2}D\left(1 - \frac{a_0}{D}\right) \tag{6}$$

$$\mathrm{p} = \frac{2}{\Gamma+2}\rho_0 D^2\left\{1 - \frac{\Gamma}{2}\left(\frac{a_0}{D}\right)^2\right\} - \frac{1}{2}\frac{(\Gamma+2)^2}{\Gamma^2}C_0\rho_0 D^2\left\{1 + \frac{2}{\Gamma}\left(\frac{a_0}{D}\right)^2\right\}^2 \tag{7}$$

$$\mathrm{h} = \frac{1}{2}\left(\frac{\Gamma+2}{\Gamma}\right)^2 C_0\rho_0 D^2\left\{1 + \frac{2}{\Gamma}\left(\frac{a_0}{D}\right)^2\right\}^2 \tag{8}$$

where $C_0 = \frac{2h_0}{\rho_0 D^2}$ is the shock Cowling number and D is the speed of the shock wave defined as $D = \frac{dR}{dt}$, since the initial energy input E_0 of explosion is very large, the shocks speed $D \gg a_0$ so that $\frac{a_0}{D} \to 0$ in the strong shock limit.

Therefore the Rankine–Hugoniot jump conditions (6)–(8) in the case of strong shock waves can be written as

$$\rho = \frac{\Gamma+2}{\Gamma}\rho_0 \qquad u = \frac{2}{\Gamma+2}D \tag{9}$$

$$p = \frac{2}{\Gamma+2}\rho_0 D^2 - \frac{1}{2}\frac{(\Gamma+2)^2}{\Gamma^2}C_0\rho_0 D^2 \tag{10}$$

$$h = \frac{1}{2}\frac{(\Gamma+2)^2}{\Gamma^2}C_0\rho_0 D^2 \tag{11}$$