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ORIGINAL ARTICLE

Rheological properties of Reiner-Rivlin fluid model for blood flow through tapered artery with stenosis

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Abstract In the present article, we have analyzed the Reiner-Rivlin fluid model for blood flow through a tapered artery with a stenosis. The constitutive equations for a Reiner-Rivlin fluid have been modeled in cylindrical coordinates. A perturbation series in dimensionless Reiner-Rivlin fluid parameter ($\lambda_1 \ll 1$) have been used to obtain explicit forms for the velocity, resistance impedance, wall shear stress and shearing stress at the stenosis throat. The graphical results of different type of tapered arteries (i.e converging tapering, diverging tapering, non-tapered artery) have been examined for different parameters of interest.

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1. Introduction

In the arterial systems of humans or animals, it is quite common to find localized narrowings, commonly called stenosis, caused by intravascular plaques. These stenosis disturb the normal pattern of blood flow through the artery [1]. Pulsatile flow of blood through a stenosed porous medium under the influence of body acceleration has been studied by El-Shahed [2]. He mentioned that the investigations of blood flow through arteries are of considerable importance in many cardiovascular diseases particularly atherosclerosis. The effects of pulsatility, stenosis and non-Newtonian behavior of blood, assuming the blood to be represented by Herschel–Bulkley fluid, are simultaneously considered by Sankara and Hemalatha [3]. Among the various arterial diseases the development of arteriosclerosis in blood vessels is quite common which may be attributed to accumulation of lipids in the arterial wall or pathological changes in the tissue structure [4].

The mathematical modeling of non-Newtonian nature of blood flow through a stenosed tube has been studied by Shukla et al. [5,6] and Chaturani and Ponnalagar Samy [7]. Blood flow in a stenosed tube has been modeled for couple stress fluid by Pralhad and Schultz [8]. Hall [9] and Porenta et al. [10] pointed out that most of the vessels could be considered as long and narrow, slowly tapering cones. Thus the effects of vessel

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tapering together with the non-Newtonian behavior of the streaming blood seem to be equally important and hence certainly deserve special attention [11,12]. Some recent studies which have been made to study the blood flow properties are cited in the refs. [13–20].

With the above motivation, an attempt is made in the present investigation to develop a mathematical model in order to study the characteristics of the Reiner Rivlin fluid model for blood flow through a tapered arteries in the presence of stenosis. The governing equations are solved analytically by regular perturbation method. The expression for velocity, resistance impedance, wall shear stress and shearing stress at the stenosis throat has been calculated. At the end, the physical features of various emerging parameters have been discussed by plotting the graphs. Trapping phenomena have been discussed at the end of the article.

2. Mathematical formulation

Let us consider an incompressible flow of Reiner Rivlin fluid having constant viscosity μ and density ρ in a tube having length L. We are considering cylindrical coordinate system (r, θ, z) such that $\bar{u}$ and $\bar{w}$ are the velocity component in $\bar{r}$ and $\bar{z}$ direction respectively. Further we assume that $r = 0$ is taken as the axis of the symmetry of the tube. The geometry of the stenosis which is assumed to be symmetric can be described as [11]

$$h(z) = d(z)[1 - \eta_1(b^{n-1}(z-a) - (z-a)^n)], \quad a \leqslant z \leqslant a + b, \tag{1}$$

$$= d(z), \quad \text{otherwise} \quad d(z) = d_0 + \xi z, \tag{2}$$

where $d(z)$ is the radius of the tapered arterial segment in the stenotic region, d_0 is the radius of the non-tapered artery in the non-stenoic region, ξ is the tapering parameter, b is the length of stenosis, $(n \geqslant 2)$ is a parameter determining the shape of the constriction profile and referred to as the shape parameter (the symmetric stenosis occurs for $n = 2$) and a indicates its location as shown in Fig. 1. The parameter η is defined as

$$\eta = \frac{\delta^* n^{\frac{n}{n-1}}}{d_0 b^n (n-1)}, \tag{3}$$

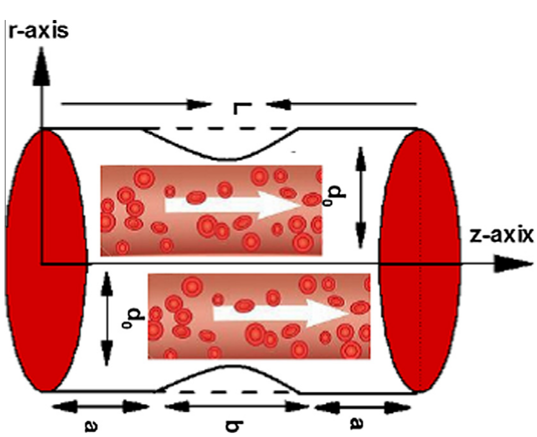

Figure 1 Geometry of an axially nonsymmetrical stenosis in the artery.

The equations governing the steady incompressible Reiner-Rivlin fluid are given as

$$\frac{\partial \bar{u}}{\partial \bar{r}} + \frac{\bar{u}}{\bar{r}} + \frac{\partial \bar{w}}{\partial \bar{z}} = 0, \tag{4}$$

$$\rho\left(\bar{u}\frac{\partial}{\partial \bar{r}} + \bar{w}\frac{\partial}{\partial \bar{z}}\right)\bar{u} = -\frac{\partial \bar{p}}{\partial \bar{r}} + \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}(\bar{r}\bar{\boldsymbol{\tau}}_{\bar{r}\bar{r}}) + \frac{\partial}{\partial \bar{z}}(\bar{\boldsymbol{\tau}}_{\bar{r}\bar{z}}) - \frac{\bar{\boldsymbol{\tau}}_{\bar{\theta}\bar{\theta}}}{\bar{r}}, \tag{5}$$

$$\rho\left(\bar{u}\frac{\partial}{\partial \bar{r}} + \bar{w}\frac{\partial}{\partial \bar{z}}\right)\bar{w} = -\frac{\partial \bar{p}}{\partial \bar{z}} + \frac{1}{\bar{r}}\frac{\partial}{\partial \bar{r}}(\bar{r}\bar{\boldsymbol{\tau}}_{\bar{r}\bar{z}}) + \frac{\partial}{\partial \bar{z}}(\bar{\boldsymbol{\tau}}_{\bar{z}\bar{z}}). \tag{6}$$

The Cauchy stress $\bar{\boldsymbol{\tau}}_{\mathbf{ij}}$ for a Reiner Rivlin fluid is given by [12]

$$\bar{\boldsymbol{\tau}}_{\mathbf{ij}} = -\bar{p}\boldsymbol{\delta}_{\mathbf{ij}} + \mu\mathbf{e}_{ij} + \mu_c\mathbf{e}_{ik}\mathbf{e}_{kj}, \quad i, j = \bar{r}, \bar{z}, \bar{\theta}, \tag{7}$$

where $\bar{\boldsymbol{\tau}}_{\mathbf{ij}}$ is the stress tensor, $\mathbf{e}_{ij}$ is the rate of strain tensor, $\boldsymbol{\delta}_{\mathbf{ij}}$ is the Kronecker delta, μ is the coefficient of viscosity and μ_c is the coefficient of cross viscosity.

We introduce the non-dimensional variables

$$r = \frac{\bar{r}}{d_0}, \; z = \frac{\bar{z}}{b}, \; w = \frac{\bar{w}}{u_0}, \; u = \frac{b\bar{u}}{u_0\delta}, \; p = \frac{d_0^2\bar{p}}{u_0 b\mu}, \; h = \frac{\bar{h}}{d_0},$$
$$\mathrm{Re} = \frac{\rho b u_0}{\mu}, \; \tilde{\boldsymbol{\tau}}_{rr} = \frac{b\bar{\boldsymbol{\tau}}_{rr}}{u_0\mu}, \; \tilde{\boldsymbol{\tau}}_{rz} = \frac{d_0\bar{\boldsymbol{\tau}}_{\mathbf{rz}}}{u_0\mu}, \; \tilde{\boldsymbol{\tau}}_{zz} = \frac{b\bar{\boldsymbol{\tau}}_{zz}}{u_0\mu}, \; \tilde{\boldsymbol{\tau}}_{\theta\theta} = \frac{b\bar{\boldsymbol{\tau}}_{\theta\theta}}{u_0\mu},$$
$$\lambda_1 = \frac{\mu_c u_0}{\mu b}, \tag{8}$$

where u_0 is the velocity averaged over the section of the tube of the width d_0.

Making use of Eqs. (7) and (8), Eqs. (4)–(6), the appropriate equations describing the steady flow of an incompressible Reiner Rivlin fluid in the case of mild stenosis $\left(\frac{\delta^*}{d_0} \ll 1\right)$, subject to the additional conditions [11] i.e

$$(i) \; \frac{\mathrm{Re}\,\delta^* n^{\left(\frac{1}{n-1}\right)}}{b} \ll 1, \quad (ii) \; \frac{d_0 n^{\left(\frac{1}{n-1}\right)}}{b} \sim O(1), \tag{9}$$

can be written as

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \tag{10}$$

$$\frac{\partial p}{\partial r} = 0, \tag{11}$$

$$\frac{\partial p}{\partial z} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\left(\left(\frac{\partial w}{\partial r}\right) + \lambda_1\left(2\frac{\partial w}{\partial r}\frac{\partial w}{\partial z}\right)\right)\right]. \tag{12}$$

The corresponding boundary conditions are

$$\frac{\partial w}{\partial r} = 0 \quad \text{at } r = 0, \; w = 0 \quad \text{at } r = h(z), \tag{13}$$

where

$$h(z) = (1 + \xi z)[1 - \eta_1((z-\sigma) - (z-\sigma)^n)], \quad \sigma \leqslant z \leqslant \sigma + 1, \tag{14}$$

and

$$\eta_1 = \frac{\delta n^{\frac{n}{n-1}}}{(n-1)}, \; \delta = \frac{\delta^*}{d_0}, \; \sigma = \frac{a}{b}, \; \xi' = \frac{\xi b}{d_0} \tag{15}$$

in which $(\xi = \tan\phi)$, ϕ is called tapered angle and for converging tapering $(\phi < 0)$, non-tapered artery $(\phi = 0)$ and the diverging tapering $(\phi > 0)$.